LINEAR AND NONLINEAR LIGHT BULLETS: RECENT THEORETICAL AND EXPERIMENTAL STUDIES

D. MIHALACHE

Horia Hulubei National Institute for Physics and Nuclear Engineering, P.O. Box MG-6,
077125 Magurele, Romania
E-mail: Dumitru.Mihalache@nipne.ro

I provide a brief overview of recent theoretical and experimental studies of unique spatiotemporal dynamics of linear and nonlinear light bullets in a variety of relevant physical settings.

1. INTRODUCTION

The past two decades have witnessed an increasing interest in both theoretical and experimental studies of shape-preserving localized structures of light, which overcome either dispersion (*temporal optical solitons*) or diffraction (*spatial optical solitons*). The temporal solitons propagating in optical fibers are pulses of a specific shape, power, and width; see, *e.g.*, the comprehensive book [1]. At wavelengths larger than 1300 nm in common optical fibers and at wavelengths larger than 1500 nm in dispersion-shifted optical fibers, the group-velocity dispersion (GVD), the sign of the relatively weak third-order (Kerr-type) optical nonlinearity, and the low optical losses in these two standard telecommunication windows, are just the right ones for long distance propagation of temporal solitons in silica glass fibers [1]. The temporal optical soliton transmission has been achieved in a commercial ultra-long-haul optical fiber network by using a dense-wavelength-division-multiplexing technique [2].

It is worthy to mention that the space-time coupling occurring when a pulsed optical beam propagates through a nonlinear medium leads to unique nonlinear effects, such as the spatiotemporal collapse for anomalous GVD of a cubic (Kerr)

nonlinear medium, pulse splitting if the GVD of the medium is normal, and the formation, in certain situations, of spatially confined light pulses (in both transverse spatial dimensions), *i.e.*, the creation of *spatiotemporal optical solitons*, alias "nonlinear light bullets"; for an early review of *nonlinear light bullets*, see Ref. [3] and some recent publications [4]-[7].

It is well known that optical solitons in media with a cubic (Kerr-like) self-focusing optical nonlinearity, obeying the nonlinear Schrödinger (NLS) equation, are unstable in two and three dimensions, because of the occurrence of optical beam collapse [8,9]. However, in the past decades, several possibilities to arrest the intrinsic wave collapse were considered. One typically uses quadratic nonlinear optical media (common second-harmonic generation materials) that support stable (robust) optical solitons regardless the physical dimension in which they form and propagate [10]-[13]. In a seminal experimental study published in 1999 by the group of F. W. Wise at Cornell University [14], formation of robust (2+1)-dimensional nonlinear light bullets was achieved in quadratic nonlinear crystals by generating the necessary anomalous GVD via the achromatic phase matching technique. Here I also mention several other physical settings, which are adequate for getting robust light bullets. These settings use (a) either saturable [15,16] or nonlocal [17,18] optical materials, (b) optical media with competing quadratic-cubic or cubic-quintic nonlinearities [19,20], (c) confining two- or three-dimensional optical lattices [21]-[25], and (d) periodic (discrete) waveguide structures with controlled diffraction and/or GVD [26]-[29].

Moreover, the formation of multidimensional fundamental (vorticityless) and spinning (vortex) dissipative solitons in media with gain and loss described by the generic cubic-quintic Ginzburg-Landau equation [30,31], and the existence of stable *discrete light bullets* in one- and two-dimensional photonic lattices [32]-[33] were also put forward in the past decade. Multidimensional localized structures have also attracted a great deal of attention in the field of atomic Bose-Einstein condensate (BEC), see, *e.g.*, Ref. [34]. In short, the *nonlinear light bullets* (see the seminal paper by Y. Silberberg [35], where the term "light bullet" was coined) are thus spatially confined pulses of light, *i.e.*, electromagnetic wave packets self-trapped in both space and time. These localized light 'balls" could be used as ideal information carriers in future all-optical processing systems.

Recently, robust and versatile localized wave packets, which combine Bessel beams in the transverse plane with temporal Airy pulses have been reported by F. W. Wise *et al.* [36]. These Airy-Bessel *linear light bullets* propagate without broadening in time or space under the combined action of dispersion and diffraction. Because the creation of such localized wave packets does not depend critically on detailed knowledge of material properties, they are truly versatile in probing light-matter interactions. Also, they could find applications ranging from bioimaging to plasma physics, see the seminal work [36].

This paper is organized as follows. I the next Section I briefly summarize the previous work on spinning (vortex) spatiotemporal solitons in different physical settings and in either conservative or dissipative systems. In Sec. 3 I concentrate on recent seminal works dealing with the first observation of three-dimensional discrete-continuous nonlinear light bullets in two-dimensional waveguide arrays [37]. Previous theoretical study [22] on the existence, stability and robustness of three-dimensional discrete-continuous spatiotemporal optical solitons in two-dimensional photonic lattices is also briefly summarized. In Sec. 4 I provide a brief overview of some selected recent theoretical and experimental works in the area of both linear and nonlinear light bullets. Finally, Sec. 5 concludes this paper.

2. ROBUST SPATIOTEMPORAL VORTEX SOLITONS IN MEDIA WITH COMPETING NONLINEARITIES

In both optics and photonics and in the exciting research area of matter waves (Bose-Einstein condensates), the localized vortices, i.e., *vortex solitons*, have drawn much attention in the past decade as objects of fundamental interest. Vortex solitons have a lot of potential applications to all-optical information processing, as well as to the guiding and trapping of atoms. The simple mathematical definition of the optical vortex is as follows: in the center of an optical vortex the electromagnetic field is equal to zero, however the circulation C of the gradient of the phase of the electromagnetic field on an arbitrary closed contour around the vortex core is a multiple of 2π, *i.e.*, C=2π S. Here the quantity S, which is an integer number, is the *topological charge* (alias topological number) of the vortex. Such vortex solitons were also called "spinning" solitons, the integer number S being the value of the "spin".

In a seminal theoretical paper [19] it was proved that stable nondissipative spatiotemporal spinning solitons (or vortex tori) with topological charge S=1 described by the three-dimensional NLS equation with focusing cubic and defocusing quintic nonlinearities were found to exist for sufficiently large energies of the vortex torus. This remarkable result also holds for the case of competing quadratic and self-defocusing cubic nonlinearities [20]. A general conclusion of these earlier theoretical studies is that stable vortex solitons are possible as a result of competition between focusing and defocusing optical nonlinearities; see Refs. [19] and [20] for more details. In Fig. 1 we show the robustness of a spatiotemporal soliton with vorticity S=1 and of relatively high energy, in media with competing nonlinearities of the cubic-quintic type, see Refs. [19] and [38]. Notice that spinning solitons with S=1 are also stable in a more general model involving bimodal cubic-quintic media (*i.e.*, in a bimodal system described by two coupled cubic-quintic NLS equations), see Ref. [38]. The solitons with "spin" S=1 can be

stable only if their energy exceeds a certain critical value, so that, in typical cases the stability region of these nonlinear vortex tori occupies about 25% of their existence domain; see Ref. [38] for a detailed study of this issue. Notice that in conservative media with competing nonlinearities all spinning solitons with topological number larger than one are unstable. However, in dissipative media all three-dimensional vortex solitons can be stable in certain domains of their relevant parameter space, see Refs. [30]-[31]. In Fig. 2 we show the typical decay of the unstable S=1 vortex spatiotemporal soliton into two stable fundamental (vorticityless) spatiotemporal solitons. Notice that the unstable S=2 vortex spatiotemporal solitons typical decay into either three or four stable fundamental spatiotemporal solitons; see Figs. 3 and 4.

The complex dynamics of two- and three-dimensional soliton clusters in optical media with competing nonlinearities has been studied too [39]-[41]. Also, robust three-dimensional soliton clusters in strongly nonlocal nonlinear media, such as three-dimensional Gaussian solitons, multipole solitons and shell solitons, have also investigated [42]. Moreover, different patterns based on both fundamental (nonspinning) solitons and vortices were theoretically investigated in optics and in the usual BEC models governed by the Gross-Pitaevskii equation with both local [43]-[45] and nonlocal nonlinearity [46].

Stable fundamental (S=0) dissipative solitons are known to exist in the generic three-dimensional model based on the cubic-quintic complex Ginzburg-Landau equation, see Refs. [47]-[50]. Moreover, three-dimensional double-soliton complexes including rotating ones were also found in this generic model of dissipative systems, see Ref. [51].

In a series of recent works, see Refs. [30,31], we performed a comprehensive stability analysis of three-dimensional dissipative solitons with intrinsic vorticity S governed by the complex Ginzburg-Landau equation with cubic and quintic terms in its dissipative and conservative parts. We found that a necessary stability condition for all vortex solitons, but not for the fundamental ones (S=0), is the presence of nonzero diffusivity in the transverse plane. The fundamental solitons are stable in all cases when they exist, while the vortex solitons are stable only in a part of their existence domain. However, the spectral filtering (*i.e.*, the temporal-domain diffusivity) is not necessary for the stability of any species of dissipative solitons, see Refs. [30] and [31] for more details. Stability domains were found for (3+1)-dimensional vortex solitons (vortex tori) with "spin" S=1, 2, and 3, suggesting that spinning solitons with any vorticity S can be stable in certain portions of their existence domains [31].

To the best of my knowledge, there is yet no experimental evidence of stable three-dimensional vortex solitons in optics. However, it is worthy to mention that in the area of fluid mechanics it was experimentally observed the formation of

toroidal viscoelastic drops, which resemble optical vortex tori; see Refs. [52, 53] for a detailed study of this remarkable phenomenon. In these works it was reported a stable toroidal drop of viscoelastic fluid sinking through a viscous Newtonian oil. In such a highly dissipative medium with a very small Reynolds number, a Newtonian vortex ring would be unstable (growing in size and slowing down), so it is the additional elasticity in the fluid that provided a sort of a stabilizing effect of these toroidal drops [52,53].

The study of localized structures in optics has recently identified *nonlinear self-similar propagation* as a robust means of avoiding optical beam or pulse breakup at high power [54]-[57]. Notice that spatiotemporally expanding self-similar light bullets and vortex tori, which are solutions of a three-dimensional NLS equation with gain were introduced recently [58]. Generation of arrays of spatiotemporal dissipative solitons by the phase modulation of a broad beam was investigated in Ref. [59], where it was proposed a simple method for obtaining stable arrays of spatiotemporal solitons in the universal three-dimensional cubic-quintic complex Ginzburg–Landau model. It is well known that a spatiotemporal pulse can be easily split by out-of-phase modulation, therefore it was imposed a periodic modulation on the broad pulse, in the transverse plane, by means of an adequately chosen phase mask. The periodic modulation splits the pulse into a cluster of spatiotemporal jets, each of them evolving into a stable spatiotemporal soliton, provided that the emerging jet has enough energy; see Ref. [59] for more details.
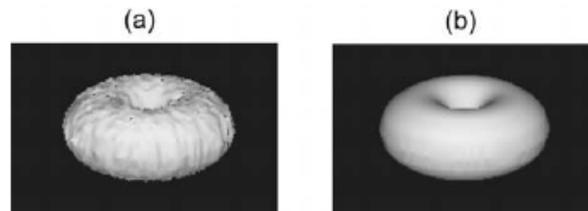

Fig. 1 – Robustness of a spatiotemporal soliton with vorticity S=1 in media with competing nonlinearities of the cubic-quintic type. Left panel: noisy input soliton. Right panel: the self-cleaned output soliton; after Ref. [38].
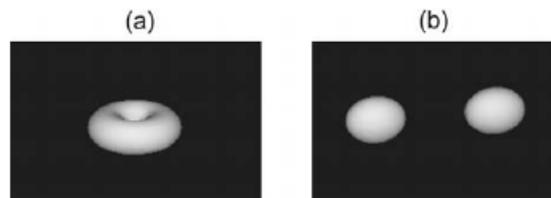

Fig. 2 – Typical decay of the unstable S=1 vortex spatiotemporal soliton into two stable fundamental spatiotemporal solitons. Left panel: input soliton. Right panel: the two output solitons; after Ref. [38].
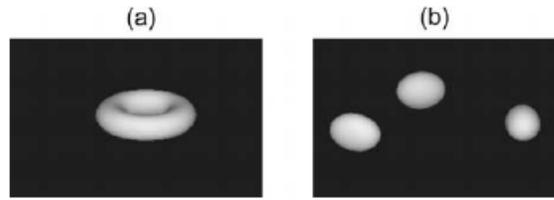
Fig. 3 – Typical decay of the unstable S=2 vortex spatiotemporal soliton into three stable fundamental spatiotemporal solitons. Left panel: input unstable soliton. Right panel: the three output solitons; after Ref. [38].
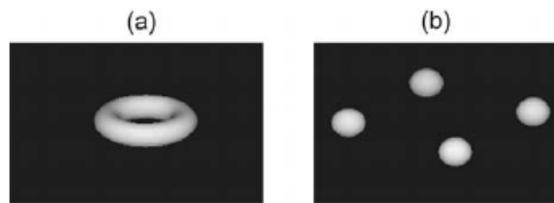

Fig. 4 – Typical decay of the unstable S=2 vortex spatiotemporal soliton into four stable fundamental spatiotemporal solitons. Left panel: input unstable soliton. Right panel: the four output solitons; after Ref. [38].

3. STABLE THREE-DIMENSIONAL CONTINUOUS-DISCRETE LIGHT BULLETS SUPPORTED BY TWO-DIMENSIONAL OPTICAL LATTICES

As was said in the Introduction the collapse in cubic (Kerr-type) focusing media can be avoided by using two-dimensional nonlinear photonic lattices in a three-dimensional setting [21]-[25]. In a seminal paper [22], it were investigated in detail both the existence and stability of three-dimensional spatiotemporal solitons in self-focusing cubic (Kerr-type) optical media with an imprinted two-dimensional harmonic transverse modulation of the refractive index. It was shown in this work that two-dimensional photonic Kerr-type nonlinear lattices can support stable one-parameter families of three-dimensional spatiotemporal solitons provided that their energy is within a certain interval and the strength p of the lattice potential, which is proportional to the refractive index modulation depth, is larger than a certain critical value p_c. As a consequence of the imprinted two-dimensional photonic lattice, the three-dimensional light bullets exist only when the nonlinear propagation constant is larger than some minimum value, which constitutes the edge of the band gap. Families of three-dimensional spatiotemporal solitons in two-dimensional harmonic lattices exist whenever their energy exceeds a certain minimum value. It was shown in Ref. [22] that three-dimensional light discrete-continuous light bullets are stable only in the intermediate-energy regime and for sufficiently large values of the two-dimensional lattice strenghth parameter p. In Fig. 5(a) we show the typical dependences of Hamiltonian versus soliton norm for

different values of the photonic lattice strength. Figure 5(b) illustrates the same dependence in the particular case $p = 0$ (for the unstable three-dimensional NLS solitons).

Remarkably, for sufficiently large values of the lattice strength parameter p, the Hamiltonian-versus-energy (soliton norm) curves display two cusps (see Fig. 5(a)), instead of a single one as in other two-dimensional and three-dimensional nondissipative (Hamiltonian) nonlinear dynamical systems. This unique two-cusp structure of the soliton norm-Hamiltonian diagram is known as the so-called *"swallowtail"* catastrophe; see, *e.g.*, Ref. [34]. Typical spatial and temporal slices of the field distribution of both stable low-amplitude discrete-continuous spatiotemporal solitons and stable high amplitude ones are shown in Fig. 6.

Figures 7 and 8 illustrate the typical soliton norm-nonlinear wave number-Hamiltonian diagram for stable three-dimensional continuous discrete light bullets confined by two-dimensional optical lattices for the value $p = 20$ of the lattice strength, and for unstable three-dimensional NLS solitons ($p = 0$), respectively. This unique bifurcation pattern is quite rare in physical systems; see Refs. [23,24] for the occurrence of such swallowtail bifurcation pattern in the study of (3+1)-dimensional light bullets supported by radially symmetric Bessel lattices [23] and in the study of (3+1)-dimensional solitons in Bose-Einstein condensates loaded in a three-dimensional lattice [24]. It is worthy to notice that such swallowtail bifurcation pattern occurs also in the study of stability of three-dimensional solitons with vorticity S=1 supported by a two-dimensional harmonic lattice if the lattice strength is large enough [25].

Recently we have introduced *discrete surface light bullets* forming in both one-dimensional [32] and two-dimensional [33] photonic lattices (for excellent reviews of discrete solitons in optics and solitons in nonlinear lattices see Ref. [60]). It is worthy to mention that the work on surface light bullets in discrete-continuous systems generalizes earlier theoretical studies of surface spatial solitons performed in the eighties and beginning of the nineties, see, *e.g.*, Refs. [61]-[64]. In Refs. [32]-[33] we analyzed spatiotemporal light localization near the edge of semi-infinite arrays of weakly coupled nonlinear optical waveguides or in the corners and the edges of two-dimensional photonic lattices and demonstrated the existence and stability (in certain regions of their existence domain) of continuous-discrete spatiotemporal surface solitons [32,33]. We have shown that their properties, such as power (energy) thresholds for their formation are strongly affected by the presence of the photonic lattice truncation.

The interactions between discrete surface light bullets were analyzed in Ref. [33]. We put forward a variety of collision scenarios and different outcomes, such as soliton fusion, soliton switching, symmetric and asymmetric scattering [33]. Seminal experimental studies of discrete-continuous nonlinear light bullets excited in two dimensional fiber arrays have been reported recently in Refs. [37] and [65].

In Ref. [65] the evolution dynamics of discrete-continuous nonlinear light bullets was investigated both experimentally and numerically. It was shown that the combination of nonlinear self-frequency shift (Raman effect), wavelength dependence of the dispersion and of the inter-waveguide coupling strength induce an adiabatic variation of the parameters of the light bullets along their propagation path, until they reach the limits of the regime of existence and decay, see Ref. [65]. The characterization of the dynamics of nonlinear light bullets was carried out by implementing a spatiotemporal, cross-correlating, spectrally resolved imaging system with femtosecond resolution. The experimental results reported in Ref. [65] are in good agreement with the numerical data if higher order nonlinear effects and the wavelength dependence of the GVD and coupling strength are included in the governing NLS model.
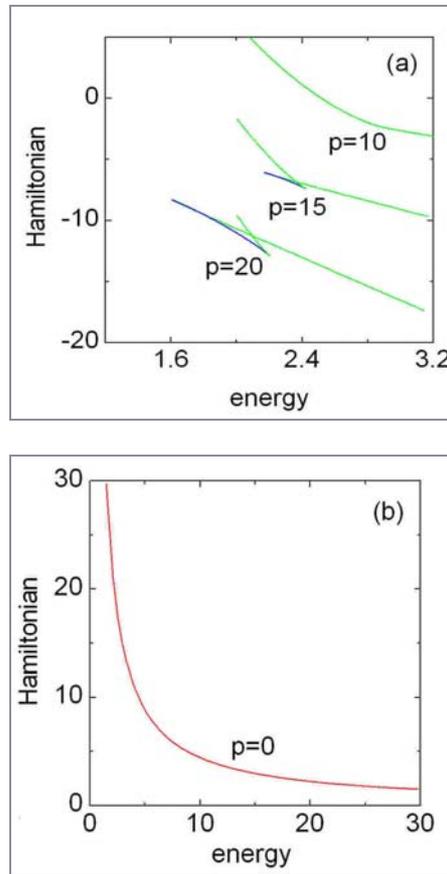

Fig. 5 – (a) Hamiltonian *versus* soliton norm for different values of the photonic lattice strength p; (b) Same as in (a) but for p=0 (the case of the unstable three-dimensional soliton described by the nonlinear Schrödinger equation); after Ref. [22].
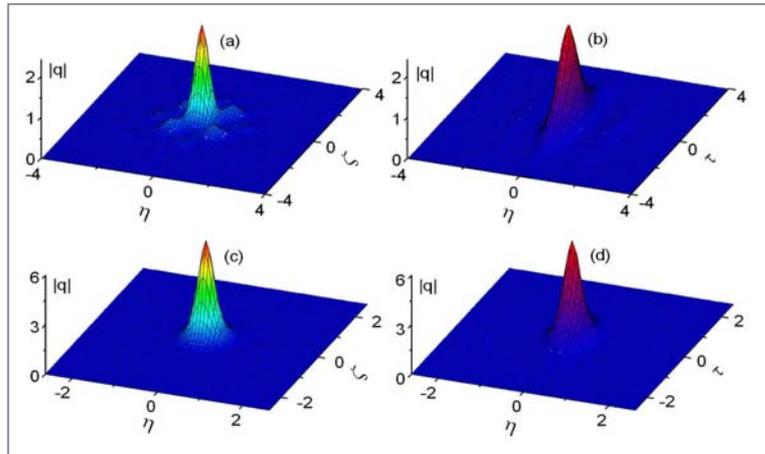
Fig. 6 ─ Typical spatial and temporal slices of the field distribution of stable low-amplitude
spatiotemporal solitons (a)-(b), and stable high amplitude spatiotemporal solitons (c)-(d);
after Ref. [22].
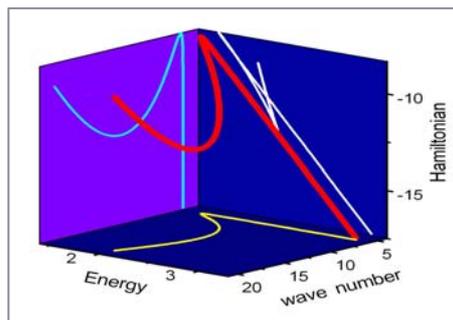

Fig. 7 – Typical soliton norm- nonlinear wave number-Hamiltonian diagram for stable three-
dimensional nonlinear light bullets confined by two-dimensional optical lattices (p=20);
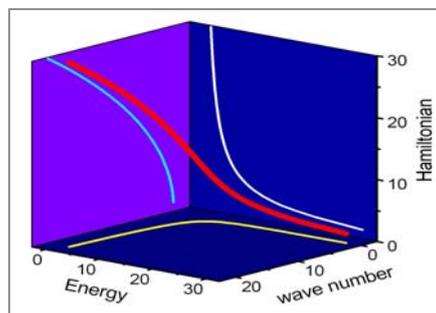after Ref. [22].

Fig. 8 – Same as in Fig. 7 but for the lattice parameter strength p=0 (the case of the unstable three-
dimensional soliton described by the nonlinear Schrödinger equation); after Ref. [22].

4. RECENT STUDIES

In this Section I briefly overview some selected recent theoretical and experimental developments in the areas of both linear and nonlinear light bullets. Trains of spatiotemporal localized structures have been recently predicted in Raman-active optical media due to spatiotemporal coupling induced by the four-wave mixing phenomenon [66]. In the majority of previous studies of nonlinear light bullet formation, the key deleterious effects of diffraction, dispersion and nonlinear self-phase modulation must be matched as a necessary condition for the creation of spatiotemporal solitons. However, recently it was put forward a new approach for realizing spatiotemporal pulse-train solitons, which is based on employing slow nonlinearities for removing the condition that the dispersion length must be equal to the diffraction length [67]. Another interesting possibility to get stable light bullets comes from the interplay of local and nonlocal nonlinearities [68]. Thus in in Ref. [68] the concept of *accessible light bullets* via synergetic optical nonlinearities (nonlocal, non-instantaneous nonlinearity combined with an instantaneous Kerr-type nonlinear response) was advanced.

A potential approach to form stable three-dimensional nonlinear light bullets might be based on the concept of engineered structures composed of different optical materials featuring either strong nonlinearity or strong suitable GVD but not necessarily both together at a given wavelength [69]. The implementation of such idea along the propagation (longitudinal) direction showed that light bullet formation is possible for significantly large tandem domains in the case of quadratic spatiotemporal solitons [69]. However, it was shown in a recent work that stable three-dimensional nonlinear light bullets do form in transverse radially periodic metamaterial structures consisting of alternating rings made of highly dispersive linear materials and rings made of strongly nonlinear media (with cubic saturable optical nonlinearities) [70].

As concerning the possible practical implementation of the light bullet concept we mention here a quite realistic physical setting involving silicon nanowires [71]. Arrays of silicon-on insulator photonic nanowires seem to be most adequate for the observation of *discrete surface light bullets* because a suitable design of optical waveguides with subwavelength transverse dimensions can provide dispersion lengths in the range of 1 mm and coupling lengths of a few millimeters (for pulse durations of the order of 100 fs) [71]. Numerical and experimental investigations of spatiotemporal nonlinear optical effects in arrays of coupled subwavelength silicon waveguides pumped with infrared femtosecond pulses leading to spectral broadening, soliton generation, and emission of Cherenkov radiation were also reported [72]. In such periodic nanostructures strong nonperturbative coupling of temporal dispersion and discrete spatial diffraction was demonstrated [72].

The study of fully three-dimensional light bullets in materials with simultaneous modulation of the linear refractive index and of the nonlinearity is of increasing interest [73] because the relevant physical settings where stable three-dimensional solitons exist are relatively rare (see the earlier overviews [4]). Moreover, since three-dimensional solitons in cubic (Kerr-like) nonlinear media exhibit the unavoidable supercritical collapse (see Refs. [8]-[9]), addition of a linear optical lattice results in their stabilization only in a certain limited range of relevant parameters. It is therefore expected that the additional modulation of the cubic optical nonlinearity may dramatically affect the domains of existence and stability of spatiotemporal optical solitons. Recently, it was investigated an interesting physical setting with competing linear and nonlinear lattices, where the refractive index is decreased in the points where the nonlinearity is strongest [73]. Thus it was investigated in Ref. [73] the stability of spatiotemporal optical solitons supported by Bessel optical lattices with out-of-phase modulation of the linear and nonlinear refractive indices. However, it has been proven that the fully three-dimensional light bullets forming in a two-dimensional Bessel lattice can be stable provided that the peak intensity does not exceed a certain critical value, see Ref. [73].

As I said before, spatiotemporal solitons forming in dissipative media (*dissipative light bullets*) have also attracted a lot of attention in the past few years. It is worthy to mention a recent work where spatiotemporal necklace-shaped patterns with annular or radial phase modulation were theoretically studied [74] in two relevant physical settings: (a) dissipative media governed by the three-dimensional complex Ginzburg-Landau equation, and (b) dissipative systems described by the three-dimensional complex Swift-Hohenberg equation. It was demonstrated that spatiotemporal necklace-shaped patterns with annular phase modulation can fuse into stable fundamental or vortex solitons in both dynamical dissipative models mentioned above, when the initial radius of the necklace is smaller than a critical value, which is similar to the fusion of two-dimensional necklace-shaped patterns into stable fundamental or vortex solitons in a similar complex Ginzburg-Landau equation, see, *e.g.*, Ref. [75]. The formation of stable dissipative light bullets in the degenerate optical parametric oscillator was also studied [76].

We recently considered continuous-discrete spatiotemporal models described by the generic complex Ginzburg-Landau equation [77]-[78]. Thus the presence of gain and loss due to optical amplifiers and saturable absorbers in truncated one- and two-dimensional periodic photonic structures has been investigated and dissipative surface light bullets were introduced in both one-dimensional waveguide arrays [77] and in two-dimensional photonic lattices [78]. The domains of existence and stability of in-phase (unstaggered) on-site (single-peaked), inter-site (double-peaked) and flat-top-like (four-peaked) dissipative light bullets in two-dimensional photonic lattices were determined and the various instability-induced scenarios of the dynamics of these discrete Ginzburg-Landau spatiotemporal

optical solitons were described [79]. Further, systematic results of collisions between discrete spatiotemporal Ginzburg-Landau solitons were reported [80]. The generic outcomes of collisions between both co-rotating [81] and counter-rotating [82] vortex solitons and between nonspinning and spinning [83] co-axial (3+1)-dimensional dissipative light bullets described by the universal complex Ginzburg-Landau equation with the cubic-quintic nonlinearity were recently reported. Depending on the value of the collision momentum χ, three generic outcomes were identified in the case of co-rotating light bullets: merger of the solitons into a single one, at small values of χ; quasielastic interaction, at large values of χ; and creation of an extra soliton, in an intermediate region, see Ref. [81]. In the case of counter-rotating light bullets, with the increase of the collision momentum, merger of the spatiotemporal vortex solitons into one or two dipole or quadrupole clusters of fundamental solitons (for S=1 and 2, respectively) is followed by the appearance of pairs of counter-rotating "unfinished vortices," in combination with a soliton cluster or without it. For relatively large values of the transverse collision momentum, the collisions become elastic. The clusters generated by the collisions are very robust, while the "unfinished vortices," eventually split into soliton pairs; see Ref. [82] for more details of this study.

Spatiotemporal solitons in the Ginzburg-Landau model with a two-dimensional transverse grating were explored in Ref. [84]. Families of spatiotemporal dissipative solitons which form in three-dimensional laser cavities including a combination of gain, saturable absorption, and transverse grating were found by numerical simulations of the governing dynamical nonlinear equations [84]. The generic model is based on the complex Ginzburg-Landau equation with the cubic-quintic nonlinearity and a two-dimensional external periodic potential representing the grating. Both fundamental (S=0) and vortical solitons were found as robust attractors in this model and their stability against strong random perturbations was tested by direct numerical simulations. It was found that the fundamental solitons are completely stable while the vortex solitons, built as rhombus-shaped complexes of four fundamental solitons, may be split by perturbations into their constituents separating in the temporal direction. Nevertheless, a sufficiently strong grating makes the vortex solitons practically stable objects, in the sense that they can be quite robust for tens of soliton's dispersion lengths; see Ref. [84] for more details.

It is also relevant to mention the study of the so-called nonlinear X-waves [85]. This is a generalization of the diffraction-free propagation of Bessel beams [86] in a linear medium to the polychromatic case. Thus it was discovered [85] that a quasi localized light pattern can spontaneously emerge from unstable propagation of a short pulse in a quadratic nonlinear crystal with normal GVD and large group- and phase-velocity mismatches. Recently, direct spatiotemporal measurements of accelerating and decelerating ultrashort Bessel-type light bullets with micron spatial resolution and femtosecond temporal resolution have been performed [87].

Localized multidimensional femtosecond optical pulses in an off-resonance two-level medium were investigated in Ref. [88] by using the quasiadiabatic following approach. It was shown that the evolution of an ultrashort pulse is governed by a generalized Kadomtsev–Petviashvili equation with coupling between the spatial and temporal profiles [88]. This work put forward the necessity of using approaches beyond the framework of the slowly-varying envelope approximation for the adequate description of spatiotemporal dynamics of femtosecond (few-cycle) spatiotemporal solitons. The existence of generic light bullets compressed to the few-cycle limit in the filamentation regime with no external compressor system has been numerically demonstrated [89]. Such few-cycle light bullets can be formed in both gaseous and dense media, see Ref. [89]. By using the powerful multiscale analysis (reductive perturbation method), a generic Kadomtsev-Petviashvili evolution equation governing the propagation of femtosecond spatiotemporal optical solitons in quadratic nonlinear media beyond the slowly-varying envelope approximation was put forward [90]. Direct numerical simulations showed the formation, from adequately chosen few-cycle input pulses, of both stable *line solitons* in the case of a quadratic medium with normal dispersion and of stable *lumps* for a quadratic medium with anomalous dispersion. Notice that the perturbed unstable line soliton typically decays into stable lumps in the case of a quadratic nonlinear medium with anomalous dispersion [90].

Collapse of ultrashort spatiotemporal pulses described by the cubic generalized Kadomtsev-Petviashvili equation was investigated in Ref. [91]. In this work, by using a reductive perturbation method, a cubic generalized Kadomtsev-Petviashvili equation for ultrashort spatiotemporal optical pulse propagation in cubic (Kerr-like) media without the use of the slowly varying envelope approximation was derived from Maxwell-Bloch equations [91]. The collapse threshold for the propagation of few-cycle spatiotemporal pulses described by the cubic generalized Kadomtsev-Petviashvili equation was calculated by a direct numerical method and it was compared to analytical results based on a rigorous virial theorem. Moreover, the typical evolution of the spectrum was given and a strongly asymmetric spectral broadening of ultrashort spatiotemporal pulses during collapse was evidenced, see Ref. [91] for a detailed discussion of this result. In a subsequent work, ultrashort light bullets in the short-wave approximation regime described by the two-dimensional sine-Gordon nonlinear evolution equation were put forward by using a reductive perturbation technique applied to a two-level model [92]. A generic (2+1)-dimensional sine-Gordon evolution equation governing the propagation of femtosecond spatiotemporal optical solitons in Kerr media beyond the slowly varying envelope approximation was investigated [92]. Direct numerical simulations showed that, in contrast to the long-wave approximation, no collapse occurs, and that robust (2+1)-dimensional ultrashort light bullets may form from adequately chosen few-cycle spatiotemporal inputs. It is worthy to notice that in contrast to the case of quadratic nonlinearity, these light

bullets oscillate in both space and time and are therefore not steady-state lump solitons; see Ref. [92] for a detailed study of this issue.

Numerical studies of the generation of stable (3+1)-dimensional high-intensity ultrashort light pulses were reported recently [93]; both self-compression of the pulse around single-cycle regime and dramatic increase of the pulse intensity were found in these extensive numerical simulations [93]. Space-time bullet trains via modulation instability and nonlocal solitons were introduced in Ref. [94]. Thus stable spatio-temporal self-trapped wavepackets (temporally dense light bullet bursts), which stem from the interplay between local electronic and nonlocal reorientational nonlinearities in nematic liquid crystals were generated by using the temporal modulational instability [94].

The concept of light bullets by *synthetic diffraction-dispersion matching* was put forward in a recent work [95]. It was shown that it is possible to generate stable, fully three-dimensional light bullets by matching the intrinsic material dispersion with a suitable effective diffraction in adequate waveguide arrays such as two-dimensional honeycomb lattices whose refractive index is periodically modulated along the direction of light propagation. By using this synthetic diffraction-dispersion matching three-dimensional light bullets were shown to form at reduced energy levels, in physical settings where the GVD would be far too weak to generate nonlinear light bullets in the absence of waveguide array, see Ref. [95] for a detailed study of this issue. Intense dynamic light bullets in a periodic lattice were also introduced [96] by using the femtosecond filamentation in air. The long propagation distance of light bullets was primarily attributed to the effect of the periodic lattice that regulates the competition between linear and nonlinear spatiotemporal effects in the region of normal dispersion. Numerical simulations reported in Ref. [96] show that it is possible to generate spatiotemporal structures inside a periodic lattice in air (a normally dispersive medium) at a wavelength of 800 nm, which remain almost invariant (in space, time and maximum intensity) for a long propagation distance of over 90 cm.

As was said in the Introduction, versatile Airy-Bessel *linear light bullets* were generated in both a silicate glass (BK7) and in a glass with large refractive index and dispersion (Schott SF14, Corning), see Ref. [10]. Such spatiotemporal optical wave packets propagate without spreading despite the joint action of dispersion and diffraction and are possible for either sign of the GVD. Moreover, they do not require equalization of diffraction and dispersion. The nonspreading nature of linear light bullets was verified for propagation through seven diffraction lengths and five dispersion lengths of the glass, *i.e.*, after passage through 15 cm of Schott SF14 glass [10]. Notice that the peak intensity of the corresponding diffracting and dispersing Gaussian pulse and beam would fall by a factor of about 250 during propagation through the same glass material. Such Airy-Bessel spatiotemporal wave packets can be used in ultrafast probing or imaging in media with poorly known or dynamically varying optical properties, see Ref. [10.]

As a natural extension of the seminal work [10], spatiotemporal Airy light bullets in both the linear and nonlinear regimes were also introduced in a recent work [97]. It was demonstrated the realization of intense Airy-Airy-Airy (Airy[3]) light bullets by combining a spatial Airy beam with an Airy pulse in time. These Airy3 light bullets are linear spatiotemporal wave packets that do not require any specific tuning of the optical properties of the material for their formation and withstand both diffraction and dispersion during their propagation. It was also shown that such light bullets maintain their robustness up to the high intensity (nonlinear) regime, resisting spreading both in space and time. Notice that the nonlinear propagation of Airy[3] light bullets was performed in a 2.5-cm-long transparent polymer sample (PMMA) for different input pulse energies [97]. The unique features of temporal and transverse spatial acceleration (bending) of these Airy[3] light bullets can be exploited in potential applications in different areas such as tomography, microscopy, long-haul data transmission systems, biophotonics, etc.; see Ref. [97].

The observation of three-dimensional discrete-continuous X-waves in photonic lattices (femtosecond laser-written waveguide arrays) has been reported recently [98]. These measurements constitute, to the best of my knowledge, the first experimental observation of temporally localized three-dimensional discrete-continuous physical objects. As was said above, fully localized (in time and two transverse spatial dimensions) three-dimensional discrete-continuous light wavepackets were observed in two subsequent recent works, see Refs. [37] and [65]. Comprehensive theoretical studies of the existence, stability and interaction scenarios of spatiotemporal solitons and vortices in optical fiber bundles have also been reported [99]. Also, spatiotemporal vortex solitons (vortical nonlinear light bullets) in hexagonal arrays of optical waveguides have been investigated in a recent work by extensive numerical simulations [100]. It was demonstrated that hexagonal lattices of parallel linearly coupled waveguides, with the intrinsic cubic self-focusing nonlinearity, give rise to three species of stable semidiscrete complexes (which are continuous in the longitudinal direction) with embedded vorticity S: (a) triangular modes with S=1, (b) hexagonal ones with topological number S=2, both centred around an empty central core, and (c) compact triangles with topological number S=1, which do not not include the empty site, see Ref. [100] for more details of this study.

Three-dimensional Bessel light-bullet solutions of the nonlinear Schrödinger equation with a photonic lattice potential in the form of squared Bessel functions in polar coordinates were studied, both analytically and numerically [101]. These light bullets appear in different forms, such as three-dimensional vortex solitons and multipole solitons, see Ref. [101]. Also, exact traveling wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional nonlinear Schrödinger equation with variable coefficients and polynomial Kerr nonlinearity of an arbitrarily high order were obtained [102]. Exact solutions, given in terms of

Jacobi elliptic functions, were presented for the cases of cubic-quintic and other relevant higher order models. It was demonstrated that the widely used method for finding exact solutions in terms of Jacobi elliptic functions is not applicable to the nonlinear Schrödinger equation with saturable nonlinearity, see Ref. [102]. Moreover, by using similarity transformations several exact solutions to three-dimensional generalized NLS equations with varying potential and nonlinearities were found [103], and exact extended travelling wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional NLS equation with distributed coefficients and linear potentials were reported in a recent publication [104].

On the theoretical arena I also mention studies on subdiffractive light pulses in photonic crystals [105], on the formation of bulletlike light pulses in photonic crystals [106], and on the propagation of self-collimated ultrashort Gaussian pulses (spatiotemporally-localized wave packets) in a hybrid photonic crystal [107]. In a recent work [107], by using the finite-difference time-domain numerical method, it was shown that ultrashort Gaussian pulses in a hybrid photonic crystal nearly retain their transverse and longitudinal shapes during propagation over a long distance, and that third-order dispersion affects the pulse shape at zero group-velocity-dispersion frequency. It is believed that these theoretical results may help to use spatiotemporally-localized wave packets in photonic devices based on photonic crystals without the presence of an optical nonlinearity; see Ref. [107]. A nanometer-scale localization of optical solitons in both space and time was recently reported by using numerical simulations of the two-dimensional two-level Maxwell-Bloch equations [108]. Thus the existence of guided optical solitons of femtosecond duration and nanometer-size mode area was put forward in Ref. [108]. The proposed physical setting exploits the confinement of light in a low refractive index subwavelength gap and the nonlinear interaction with a resonantly absorbing medium to achieve stable and ultraconcentrated coherent pulses [108].

In connection to these many theoretical studies performed in the past few years on light bullet formation in different physical setting we briefly discuss here the relevance and importance of a recent experimental work on the dynamics of spatiotemporal nonlinear localization in arrays of evanescently coupled silica fiber arrays, which form two-dimensional waveguide lattices, see Refs. [37] and [65]. Thus it was shown that in contrast to continuous systems the formation of *stable nonlinear light bullets* becomes possible, see Refs. [37] and [65]. It was demonstrated experimentally that light pulses injected into such waveguide arrays form robust solitons in both space and time, *i.e.*, nonlinear light bullets. In order to stabilize the (3+1)-dimensional spatiotemporal solitons it was exploited the discreteness of the two-dimensional photonic lattice, which had been considered theoretically in the earlier work [22]. This seminal experimental study was performed by launching 170-fs pulses at 1550-nm wavelength into a single waveguide in the centre of the photonic lattice made of 91 silica cores embedded in a fluorine-doped silica glass, and recording the spatial and temporal profiles of the

pulses at the output end of this periodic structure. Notice that at this infrared wavelength, the glass has anomalous GVD, a necessary condition for the existence of bright spatiotemporal solitons. It was concluded in this experiment that the (3+1)-dimensional nonlinear light bullets propagate stably in the interior of the waveguide array because the spatial profile was stationary for about twice as far as it would be in linear propagation, and the temporal profile was stationary for about nine times as far as it would be in linear propagation. However, the unavoidable Raman scattering in the hexagonal array of silica glass waveguides leads to a redshift of the pulse spectrum during propagation and correspondingly to an abrupt light bullet decay, because this redshift pulls the light bullet outside the domain of its existence, see Refs. [37] and [65]. Notice that in the recent work [65] the maximum possible range of existence of discrete-continuous light bullets in arrays of silica waveguides was discussed in much detail. Such (3+1)-dimensional spatiotemporal optical solitons are ideal bits of information; they offer potential for ultrafast optical digital logic, with very high switching rates of the order of several terahertz.

5. CONCLUSIONS

In this work I have attempted to provide the interested reader with a general overview of the current state of art of the growing research area of linear and nonlinear light bullets which form in a variety of physical settings. I conclude with the hope that this brief overview on recent exciting theoretical and experimental developments in the area of light bullets will inspire further studies. Finally, I do believe that interesting and exciting times have arrived for this research topic.

REFERENCES

1. L. F. Mollenauer, J. P. Gordon, *Solitons in Optical Fibers: Fundamentals and Applications*, Elsevier-Academic Press, 2006.
2. J. McEntee*, Solitons go the distance in ultra-long-haul dense-wavelength-division-multiplexing*, Fibre Systems Europe, January 19–21 (2003).
3. B. A. Malomed *et al*., J. Opt. B: Quantum Semiclassical Opt., **7**, R53 (2005);
   D. Mihalache, J. Optoelectronics Adv. Mat., **12**, 12 (2010);
   D. Mihalache, J. Optoelectronics Adv. Mat., **13**, 1055 (2011).
4. D. Mihalache, Rom. Rep. Phys., **62**, 99 (2010);
   D. Mihalache, D. Mazilu, Rom. Rep. Phys., **60**, 749 (2008);
   D. Mihalache, D. Mazilu, Rom. Rep. Phys., **60**, 957 (2008).
5. D. Mihalache, Rom. Rep. Phys., **62**, 697 (2010).
6. D. Mihalache, Proc. Romanian Acad. A, **11**, 142 (2010).

7. D. Mihalache, Rom. Rep. Phys., **63**, 9 (2011); D. Mihalache, Rom. Rep. Phys., **63**, 325 (2011).
8. L. Berge, Phys. Rep., **303**, 260 (1998).
9. L.-C. Crasovan *et al*., Phys. Rev. Lett., **91**, 063904 (2003).
10. N.-C. Panoiu *et al*., Phys. Rev. E, **71**, 036615 (2005).
11. L. Torner *et al*., Phys. Rev. Lett., **77**, 2455 (1996);
    D. Mihalache *et al*., Phys. Rev. Lett., **81**, 4353 (1998);
    D. Mihalache *et al*., Opt. Commun., **169**, 341 (1999).
12. D. Mihalache, D. Mazilu, B. A. Malomed, L. Torner, Opt. Commun., **152**, 365 (1998).
13. D. Mihalache et al., Phys. Rev. E, **62**, 7340 (2000).
14. X. Liu, L. J. Qian, F. W. Wise, Phys. Rev. Lett., **82**, 4631 (1999).
15. D. E. Edmundson, R. H. Enns, Opt. Lett., **17**, 586 (1992).
16. N. Akhmediev, J. M. Soto-Crespo, Phys. Rev. A, **47**, 1358 (1993).
17. O. Bang, W. Krolikowski, J. Wyller, J. J. Rasmussen, Phys. Rev. E, **66**, 046619 (2002).
18. D. Mihalache *et al*., Phys. Rev. E, **73**, 025601 (2006).
19. D. Mihalache *et al*., Phys. Rev. Lett., **88**, 073902 (2002).
20. D. Mihalache *et al*., Phys. Rev. E, **66**, 016613 (2002).
21. B. B. Baizakov, B. A. Malomed, M. Salerno, Phys. Rev. A, **70**, 053613 (2004).
22. D. Mihalache *et al*., Phys. Rev. E, **70**, 055603 (2004).
23. D. Mihalache *et al*.,, Phys. Rev. Lett., **95**, 023902 (2005).
24. D. Mihalache *et al*., Phys. Rev. A, **72**, 021601 (2005).
25. H. Leblond *et al*., Phys. Rev. E, **76**, 026604 (2007).
26. A. B. Aceves, C. De Angelis, A. M. Rubenchik, S. K. Turitsyn, Opt. Lett., **19**, 329 (1994);
    A. V. Buryak, N. N. Akhmediev, IEEE J. Quantum Electron., **31**, 682 (1995).
27. E. W. Laedke, K. H. Spatschek, S. K. Turitsyn, Phys. Rev. Lett., **73**, 1055 (1994).
28. Z. Xu et al., Phys. Rev. E, **70**, 066618 (2004).
29. A. A. Sukhorukov, Y. S. Kivshar, Phys. Rev. Lett., **97**, 233901 (2006).
30. D. Mihalache *et al*., Phys. Rev. Lett., **97**, 073904 (2006);
    L.-C. Crasovan *et al.,* Phys. Lett. A, **289**, 59 (2001);
    L.-C. Crasovan *et al*., Phys. Rev. E, **63**, 016605 (2001);
    D. Mihalache *et al.*, Phys. Rev. A, **82**, 023812 (2010);
    V. Skarka *et al*., Phys. Rev. Lett., **105**, 213901 (2010).
31. D. Mihalache *et al*., Phys. Rev. A, **75**, 033811 (2007); Phys. Rev. A, **76**, 045803 (2007).
32. D. Mihalache, D. Mazilu, F. Lederer, Y. S. Kivshar, Opt. Express, **15**, 589 (2007);
    Opt. Lett., **32**, 3173 (2007).
33. D. Mihalache, D. Mazilu, F. Lederer, Y. S. Kivshar, Phys. Rev. A, **79**, 013811 (2009);
    Opt. Commun., **282**, 3000 (2009).
34. D. Mihalache, D. Mazilu, Rom. Rep. Phys., **60**, 957 (2008); B. A. Malomed *et al*., Phys. Lett. A,
    361, 336 (2007)
35. Y. Silberberg, Opt. Lett., **15**, 1282 (1990).
36. A. Chong, W. H. Renninger, D. N. Christodoulides, F. W. Wise, Nat. Photon., **4**, 103 (2010).
37. S. Minardi *et al*., Phys. Rev. Lett., **105**, 263901 (2010).
38. D. Mihalache *et al*., Phys. Rev. E, **67**, 056608 (2003).
39. Y. V. Kartashov *et al*., Phys. Rev. Lett., **89**, 273902 (2002);
    D. Mihalache *et al*., Phys. Rev. E, **68**, 046612 (2003).
40. L.-C. Crasovan *et al*., Phys. Rev. E, **67**, 046610 (2003).
41. D. Mihalache *et al*., J. Opt. B: Quantum Semiclass. Opt., **6**, S333 (2004).
42. Wei-Ping Zhong *et al*., J. Phys. B: At. Mol. Opt. Phys., **41**, 025402 (2008).
43. L.-C. Crasovan *et al*., Phys. Rev. E, **66**, 036612 (2002).
44. L.-C. Crasovan *et al*, Phys. Rev. A, **68**, 063609 (2003).
45. D. Mihalache, D. Mazilu, B.A. Malomed, F. Lederer, Phys. Rev. A, **73**, 043615 (2006).
46. Y. J. He *et al*., Phys. Rev. A, **77**, 043826 (2008); Phys. Rev. A, **78**, 023824 (2008).
47. P. Grelu *et al*., Opt. Express, **13**, 9352 (2005).

48. V. Skarka, N. B. Aleksić, Phys. Rev. Lett., **96**, 013903 (2006).
49. J. M. Soto-Crespo *et al*., Opt. Express, **13**, 4013 (2006).
50. N. Akhmediev *et al*., Chaos, **17**, 037112 (2007).
51. J. M. Soto-Crespo, N. Akhmediev, P. Grelu, Phys. Rev. E, **74**, 046612 (2006).
52. M. C. Sostarecz, A. Belmonte, J. Fluid Mech., **497**, 235 (2003).
53. M. C. Sostarecz, A. Belmonte, Phys. Fluids, **15**, S5 (2003).
54. F. O. Ilday, J. R. Buckley, W.G. Clark, F. W. Wise, Phys. Rev. Lett., **92**, 213902 (2004).
55. S. A. Ponomarenko, G. P. Agrawal, Phys. Rev. Lett., **97**, 013901 (2006).
56. J. M. Dudley, C. Finot, D. J. Richardson, G. Millot, Nature Phys., **3**, 597 (2007).
57. J. Belmonte-Beitia *et al.,* Phys. Rev. Lett., **100**, 164102 (2008).
58. S. Chen, J. M. Dudley, Phys. Rev. Lett., **102**, 233903 (2009).
59. Yingji He *et al*., J. Opt. Soc. Am. B, **27**, 1266 (2010).
60. F. Lederer et al., Phys. Rep., **463**, 1 (2008);
    Yu. S. Kivshar, Laser Phys. Lett., **5**, 703 (2008);
    Y. V. Kartashov, B. A. Malomed, L. Torner, Rev. Mod. Phys., **83**, 247 (2011).
61. V. K. Fedyanin, D. Mihalache, Z. Phys. B, **47**, 167 (1982);
    D. Mihalache, R. G. Nazmitdinov, V. K. Fedyanin, Physica Scripta, **29**, 269 (1984);
    D. Mihalache, M. Bertolotti, C. Sibilia, Prog. Opt., **27**, 229 (1989).
62. G. I. Stegeman *et al*., Appl. Phys. Lett., **44**, 830 (1984).
63. N. Akhmediev, V. I. Korneyev, Y. V. Kuzmenko, Zh. Eksp. Teor. Fiz., **88**, 107 (1985) [Sov. Phys. JETP, **61**, 62 (1985).
64. F. Lederer *et al*., Opt. Commun., **99**, 95 (1993).
65. F. Eilenberger *et al*., Phys. Rev. A, **84**, 013836 (2011).
66. A. V. Gorbach, D. V. Skryabin, Phys. Rev. Lett., **98**, 243601 (2007).
67. H. C. Gurgov, O. Cohen, Opt. Express, **17**, 7052 (2009).
68. I. B. Burgess *et al*, Phys. Rev. Lett., **102**, 203903 (2009).
69. L. Torner *et al*., Opt. Commun., **199**, 277 (2001).
70. L. Torner, Y. V. Kartashov, Opt. Lett., **34**, 1129 (2009).
71. C. J. Benton *et al*., Phys. Rev. A, **78**, 033818 (2008).
72. A. V. Gorbach *et al*., Phys. Rev. A, **82**, 041802 (2010).
73. F. Ye, Y. V. Kartashov, B. Hu, L. Torner, Opt. Express, **17**, 11328 (2009).
74. Bin Liu *et al*., Opt. Express, **17**, 12203 (2009).
75. Y. J. He, H. Z. Wang, B. A. Malomed, Opt. Express, **15**, 17502 (2007).
76. N. Veretenov, M. Tlidi, Phys. Rev. A, **80**, 023822 (2009).
77. D. Mihalache *et al*., Phys. Rev. A, **77**, 043828 (2008).
78. D. Mihalache *et al*., Phys. Rev. E, **78**, 056602 (2008).
79. D. Mihalache *et al*., Eur. Phys. J. Special Topics, **173**, 255 (2009).
80. D. Mihalache *et al*., Eur. Phys. J. Special Topics, **173**, 267 (2009).
81. D. Mihalache *et al*., Phys. Rev. A, **77**, 033817 (2008).
82. D. Mihalache *et al*., Phys. Rev. E, **78**, 056601 (2008).
83. D. Mihalache *et al*., Eur. Phys. J. Special Topics, **173**, 245 (2009).
84. D. Mihalache *et al*., Phys. Rev. A, **81**, 025801 (2010).
85. P. Di Trapani *et al*., Phys. Rev. Lett., **91**, 093904 (2003).
86. P. Saari, K. Reivelt, Phys. Rev. Lett., **79**, 4135 (1997).
87. H. Valtna-Lukner et al., Opt. Express, **17**, 14948 (2009).
88. I. V. Mel'nikov, D. Mihalache, N.-C. Panoiu, Opt. Commun., **181**, 345 (2000).
89. L. Bergé, S. Skupin, Phys. Rev. Lett., **100**, 113902 (2008).
90. H. Leblond *et al*., Phys. Rev. A, **80**, 053812 (2009).
91. H. Leblond *et al*., Phys. Rev. A, **81**, 033824 (2010).
92. H. Leblond, D. Mihalache, Phys. Rev. A, **81**, 063815 (2010);
    H. Leblond, D. Mihalache, Rom.Rep.Phys., **63**, 1254 (2011).
93. T. P. Todorov *et al*., AIP Conf. Proc., **1301**, 587 (2010).

94. M. Peccianti *et al*., Opt. Express, **18**, 5934 (2010).
95. V. E. Lobanov, Y. V. Kartashov, L. Torner, Phys. Rev. Lett., **105**, 033901 (2010).
96. P. Panagiotopoulos *et al*., Opt. Express, **19**, 10057 (2011).
97. D. Abdollahpour *et al*., Phys. Rev. Lett., **105**, 253901 (2010).
98. M. Heinrich *et al*., Phys. Rev. Lett., **103**, 113903 (2009).
99. H. Leblond *et al*., Phys. Rev. A, **77**, 063804 (2008); Phys. Rev. A, **79**, 033841 (2009).
100. H. Leblond *et al*, Phys. Rev. A, **83**, 063825 (2011).
101. Wei-Ping Zhong, Milivoj Belić, Tingwen Huang, Phys. Rev. A, **82**, 033834 (2010).
102. Nikola Z. Petrović, Milivoj Belić, Wei-Ping Zhong, Phys. Rev. E, **83**, 026604 (2011).
103. Z. Y. Yan, V. V. Konotop, Phys. Rev. E, **80**, 036607 (2009).
104. N. Z. Petrović, H. Zahreddine, M. R. Belić, Physica Scripta, **83**, 065001 (2011).
105. K. Staliunas, C. Serrat, R. Herrero, C. Cojocaru, J. Trull, Phys. Rev. E, **74**, 016605 (2006).
106. C. Zhou, Q. Gong, P. Yao, D. Zhao, X. Jiang, Appl. Phys. Lett., **93**, 061103 (2008).
107. K. B. Chung, Opt. Express, **19**, 15705 (2011).
108. A. Pusch, J. M. Hamm, O. Hess, Phys. Rev. A, **84**, 023827 (2011).