What Can We Learn from the Instabilities of Asymmetric Nuclear Matter?

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Abstract

Based on a general approach to binary systems we show that in low density region asymmetric nuclear matter (ANM) is unstable only against isoscalarlike fluctuations. The physical meaning of the thermodynamical chemical and mechanical instabilities is related to the inequality relations verified by the strength of interaction among different components. Relevance of these results for bulk and neck fragmentation in the reaction $^{1}24$Sn + $^{1}24$Sn at 50 MeV/n is discussed.

1 Introduction

The availability of exotic beams has driven a large interest on nuclear structure studies of β-unstable nuclei. Essential complementary information can come from charge asymmetry effects on non-equilibrium nuclear dynamics. Even if ANM models at high density have been tested so far only in astrophysical context, supernovae explosions and neutron stars, it could be possible to extract some unique information on the symmetry term of the nuclear equation of state (EOS) in regions away from normal density also under laboratory controlled conditions.

In heavy ion collisions at intermediate energies an ubiquitous phenomena is the multifragmetation process which display several features analogous to usual liquid-gas phase transition. However in this analogy one should be aware of differences due to Coulomb, finite-size and quantum effects. For semi-central collisions in the participant zone an unstable, neck-like structure, is formed. Mid-rapidity fragment emission and the fast fission of the spectators were observed.

Here we will focus our discussion on the new features induced by the binary, i.e. two-component character of nuclear matter, both in multifragmentation as well as in neck fragmentation.

2 Instabilities and fluctuations in ANM

A binary system, including ANM, manifest a richer thermodynamical behaviour as a consequence of the new degree of freedom. The phase transitions are more complex because they have to accommodate one more conservation law. Also a variety of thermal fluctuations with different components composition can develop in the system.

In particular, in symmetric nuclear matter (SNM) one encounters two kinds of density fluctuations: (i) isoscalar, when the densities of the two components oscillate in phase with equal amplitude; (ii) isovector when the two densities fluctuate still with equal amplitude but out of phase. These are the normal modes of the system in the sense that the variation of free energy density up to second order reduces to a normal quadratic form when is expressed in terms of them. Mechanical instability is associated with instability against isoscalar fluctuations leading to cluster formation while chemical instability is related to instability against isovector fluctuations, leading to species separation.

A general analysis, for ANM, can be performed in the framework of the Fermi liquid theory [1]. We limit ourselves to monopolar deformation for proton and neutron Fermi seas ν_p^0, ν_n^0 and consider here momentum independent interactions such that $F_{l=0}^{q_1 q_2}$ ($q_i = n, p$) is the only non-zero Landau parameter. Then the thermodynamical stability condition at $T = 0$ is obtained as:

$$\delta H - \mu_p \delta\rho_p - \mu_n \delta\rho_n = \frac{1}{2}(a{\nu_p^0}^2 + b{\nu_n^0}^2 + c\nu_p^0\nu_n^0) > 0 \tag{1}$$

where H is the energy density and

$$\begin{aligned} a = N_p(0)(1+F_0^{pp}) \ \ ; \ b = N_n(0)(1+F_0^{nn}) \ \ ; \\ c = N_n(0)F_0^{pn} + N_p(0)F_0^{np}. \end{aligned} \tag{2}$$

with N_q the single-particle level density at the Fermi energy We diagonalize the form of Eq.(1) by introducing the following transformation:

$$\begin{aligned} u &= cos\beta \ \nu_p^0 + sin\beta \ \nu_n^0, \\ v &= -sin\beta \ \nu_p^0 + cos\beta \ \nu_n^0, \end{aligned} \tag{3}$$

where a *mixing* angle $0 \leq \beta \leq \pi/2$ is defined by

$$tg \ 2\beta = \frac{c}{a-b} = \frac{N_n(0)F_0^{pn} + N_p(0)F_0^{np}}{N_p(0)(1+F_0^{pp}) - N_n(0)(1+F_0^{nn})}. \tag{4}$$

Then Eq.(1) takes the form

$$\delta H - \mu_p \delta\rho_p - \mu_n \delta\rho_n = Xu^2 + Yv^2 \tag{5}$$

where

$$X = \frac{1}{2}(\ a + b + sign(c)\sqrt{(a-b)^2 + c^2} \ ) \equiv \frac{N_p(0)+N_n(0)}{2}(\ 1 + F_{0g}^s \ ) \tag{6}$$

and similar

$$Y = \frac{1}{2}(\ a + b - sign(c)\sqrt{(a-b)^2 + c^2} \ ) \equiv \frac{N_p(0)+N_n(0)}{2}(\ 1 + F_{0g}^a \ ). \tag{7}$$

Thus with the rotation (3) we separate the total variation Eq.(1) into two independent contributions, the new "normal" modes, characterized by a "mixing angle" β, which depends on the density of states and the interaction. since

$cos\beta, sin\beta$ are both positive, we interpret u- and v-variations as new independent *isoscalar*-like and *isovector*-like fluctuations appropriate for asymmetric systems. The proton and neutron densities will fluctuate in phase for isoscalar-like variations and out of phase for isovector-like variations, see Eq.(3) and they reduces to the usual isoscalar and isovector modes for SNM.

From Eq.(5) we see that thermodynamical stability requires $X > 0$ *and* $Y > 0$. Equivalently, the following conditions have to be fulfilled:

$$1 + F^s_{0g} > 0 \quad and \quad 1 + F^a_{0g} > 0, \tag{8}$$

They represent Pomeranchuk stability conditions extended to asymmetric binary systems.

The new stability conditions, Eq.(8), are equivalent to mechanical and chemical stability of a thermodynamical state, [2], [3], [4], i.e.

$$\left(\frac{\partial P}{\partial \rho}\right)_{T,y} > 0 \quad and \quad \left(\frac{\partial \mu_p}{\partial y}\right)_{T,P} > 0 \tag{9}$$

where P is the pressure and y the proton fraction, see [6] . Such general analysis leads to the conclusion that in the low energy region, where $c < 0$ as predicted by all effective interactions, the asymmetric nuclear matter is unstable only against isoscalarlike modes. This is shown for a Skyrme-like interaction [6] in Figure 1, but this effect is expected to by very robust, present for all interactions.

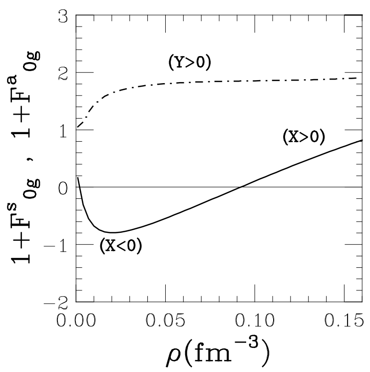

Figure 1: Density dependence of generalized Landau parameters for y=0.1

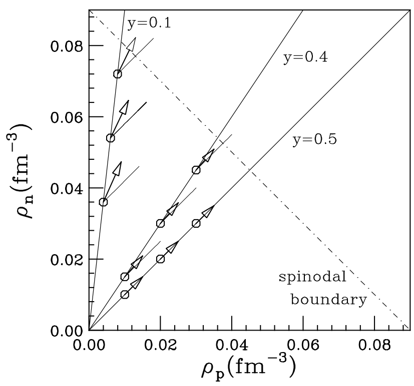

Figure 2: The unstable normal modes for liquid phase in the spinodal region.

The direction of unstable mode in the plane $\rho_n - \rho_p$ in the liquid phase is indicated by the arrows in the Figure 2. The arrow picks toward less asymmetric liquid phase: the isospin distillation take place. In this figure the length of the arrows was drawn to correspond to the same proton density perturbation. Therefore it indicate the "response" of neutrons and their relative sizes allows a comparison of the effect throughout spinodal region. In the gas phase the asymmetry is larger then the initial value, the arrows being oriented in opposite directions. The tilting angle is a measure of isospin distillation which will depend on the slope of the symmetry energy in the low energy region. Therefore the isospin content of the fragments will be an interesting observable to constrain the isovector part of the EOS, see the next sections.

The physical meaning of the thermodynamical chemical and mechanical instabilities is connected to the inequality relations existing between the interactions among different species [6]. Indeed the system will show up as a chemical instability if $(-ta - b/t) < c < -2\sqrt{ab}$ and as a mechanical instability if $c < (-ta - b/t) < -2\sqrt{ab}$, where $t = \frac{y}{1-y}\frac{N_n(0)}{N_p(0)}$. The regions were each of these conditions are verified are indicated in Figure 3 as chemical and mechanical instability respectively.

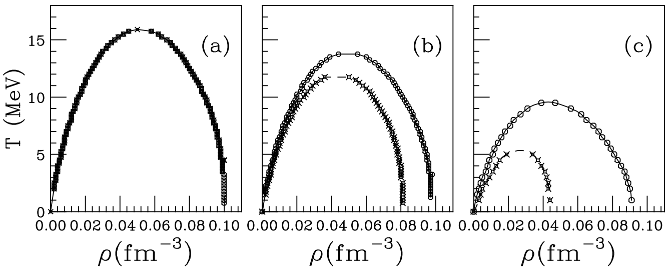

Figure 3: Spinodal boundary of asymmetric nuclear matter(open circles) and mechanical instability boundary (crosses) for three proton fractions:(a) $y = 0.5$,(b) $y = 0.25$,(c) $y = 0.1$

3 Kinetic of Phase Transition in ANM

Contrary to the situation regarding the phase transition in macroscopic systems were the observation time scale is much larger than the time scale of microscopic process that leads to drops (bubbles) formation (except for the critical point where are required days because of slowing dawn phenomenon) for the heavy ions collision the reaction time can be comparable to the fragment formation time. The violent collision may quench the system in the instability region of phase diagram. The kinetic mechanism responsible for the phase transition is the spinodal decomposition. The unstable fluctuations are amplified and growth exponentially in the first stages leading to higher density domains. In asymmetric nuclear matter they are especially interesting because of isoscalarlike character of the unstable mode and presence of isospin distillation.

We have studied the spinodal decomposition for ANM in a box with side $L = 24fm$ for several values of the initial asymmetry and initial density at temperature $T = 5$ MeV. Several kinds of initial perturbation were created automatically due to the random choice of particle positions. Results for the initial asymmetry $I = 0.5$ and initial conditions corresponding to chemical instability region ($\rho_{init} = 0.09fm^{-3}$) are reported in Figure 4 (a) and (b) respectively. It is shown the time evolution of neutron (thick histogram) and proton (thin histogram) abundance Figure 4 (a) and asymmetry ,Figure 4 (b), in various density bins. The dashed line respectively shows the initial uniform density value $\rho \simeq 0.6\rho_0$ and the initial asymmetry $I = 0.5$. The in-phase driving to higher density for neutrons and protons is evident indicating the isoscalarlike character of the instability. The consequence is the fragment formation. However the clustering mechanism is accompanied by isospin distillation leading to very different asymmetries, lower (higher) in the liquid (gas) phases.

We want to stress that an identical qualitative behaviour is observed if we start from the mechanical instability region [5], in agreement to the discussion of previous section.

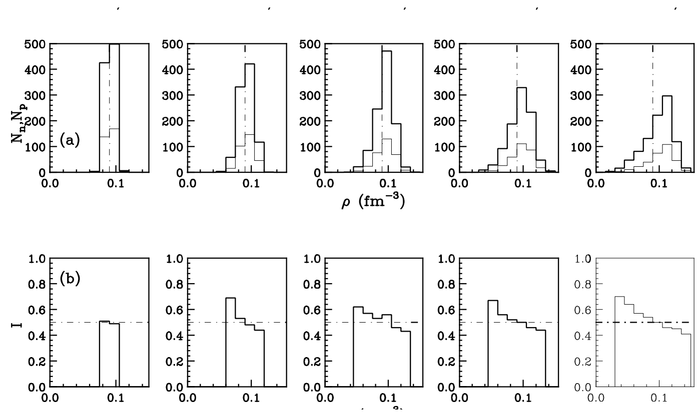

Figure 4: Time dependence of the proton and neutron numbers (a) and asymmetry (b) in different density bins.

4 Isospin dynamics from bulk to neck fragmentation

We follow the reaction dynamics for the collision of the system $^{124}Sn +^{124} Sn$ at 50AMeV for two impact parameters $b = 2fm$ and $b = 6fm$ respectively. We consider an asystiff EOS corresponding to a linear dependence with the density of the mean-field symmetry term.

For a semicentral collision, $b = 2fm$, the reaction mechanism corresponds to bulk fragmentation. After a first compression phase (until about 40-50fm/c) a fast expansion phase follows (until 110-120fm/c). Then during the fragmentation stage the system will break up and the fragment formation process take place up to the freeze-out time (around 260-280fm/c). These three main stages are characterized by specific features of the isospin dynamics since the system explores different density regions.

In particular we focused on the time evolution of the asymmetry of the "central" gas (solid line and squares), and total gas (dashed+squares), as well as of the "central" liquid (solid+circles) and IMF (clusters with $3 < Z < 23$, stars), 5 (a). By "central" we mean a region having a linear dimension of the order of $20fm$ corresponding to the active volume in which the fragmentation take place.

The spinodal mechanism is well evidenced by the inversion in the trend for both liquid and gas phase corresponding to isospin distillation. The isospin

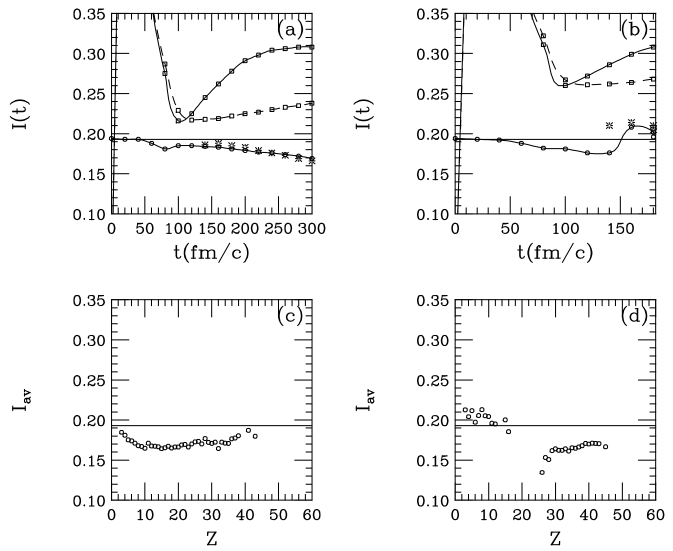

Figure 5: Time dependence of the asymmetry in gas and liquid phases for b=2fm (a) and b=6fm (b) as well as the asymmetry of the primary fragments at freeze-out time for b=2fm (c) and b=6fm (d).

content of primary fragments is below the initial value but also clearly lower than the value at the beginning of the phase transition, due to isospin distillation, see Figure 5 (a) and (c).

For b=6fm we observe a quite different behaviour. Now in the overlap region a neck structure is developing. During the interaction time (from about 80-120fm/c) it heats and expands but remains in contact with the denser and colder region of projectile-like (PLF) and target-like (TLF). Now the surface instabilities of a cylindrical shape neck region and the fast leading motion of the PLF and TLF will play an important role in the mechanism of IMF production. A correspondingly different dynamics of the isospin is evident from the Figure 5 (b). A larger asymmetry of IMF, Figure 5 (d), suggest a neck region richer (poorer) in neutrons (protons). This can be related to proton and neutron migrations between more diluted neck region and denser PLF and TLF as dictated by the chemical potential gradients of the two species.

5 Conclusion

In the previous section in an approach based on a Fermi liquid theory for two components systems we analyze several properties of ANM in low density region of phase diagram. It is concluded that:

-in the low density region, of interest for nuclear liquid-gas phase transition, the system can be characterized by a **unique spinodal region**, see also [6], defined by the instability against isoscalarlike fluctuations; inside this we can identify a mechanical instability region and a chemical one;

-the physical meaning of chemical and mechanical instabilities is connected to the inequality relations which are satisfied by interactions among different components; when this inequality changes the sign the chemical instability transform in mechanical one;

-during the time development of the spinodal decomposition in ANM the fragment formation accompanied by the isospin distillation is observed. The qualitative behaviour is unchanged when we move the initial conditions from chemical instability region to mechanical instability one;

We have also made a connection of these features with isospin transport properties in microscopic simulations of fragmentation reactions of charge asymmetric ions in the medium energy range. We observed a different isospin dynamics when we pass from bulk fragmentation, for semi-central collisions to neck fragmentation, in semi-peripheral collision.

We also mention that a dependence on the symmetry term of EOS was evidenced for the observable showed above as well as for some other as is discussed in more detail in the refs. [7], [8], [9], [10].

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