FINE STRUCTURE OF RADIATION SPECTRUM OF CHARGED PARTICLES MOVING IN MAGNETIC FIELDS IN NONABSORBABLE ISOTROPIC MEDIA AND IN VACUUM

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Integral expressions for spectral distributions of the radiation power for systems of non-interacting point charged particles moving on arbitrary trajectory in electromagnetic fields in nonabsorbable media and in vacuum are obtained using the Lorentz's self-interaction method. The fine structure of spectra of synchrotron, Cherenkov, and synchrotron-Cherenkov radiations of electrons moving in a spiral are investigated with analytical and numerical methods.

Key words: Cherenkov radiation, synchrotron radiation, synchrotron-Cherenkov radiation, Lorentz's self-interaction.

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1. Introduction

The investigations of radiation spectra of systems of charged particles moving along an arbitrary trajectory in electromagnetic fields in nonabsorbable isotropic medium and in vacuum are important from the point of view of their applications in astrophysics, electronics, plasma physics etc. [1-3]. In a case of charged particles moving in a spiral in a constant magnetic field the fine structure calculations of radiation spectra can also be used in applications.

The aim of this paper is to use the Lorentz's self-interaction method to investigate the spectral distribution of the radiation power for a system of charged particles moving along an arbitrary trajectory in electromagnetic fields in isotropic nonabsorbable media and in vacuum. A great attention was given to the Doppler effect influence on peculiarities of the radiation spectrum of a charged particle moving in a spiral in a medium and in vacuum. Using the exact integral relations for the spectral distribution of radiation power, the fine structure of the spectra of synchrotron, Cherenkov, and synchrotron-Cherenkov radiations were investigated by means of analytical and numerical methods.

2.Instantaneous and time-averaged radiation powers of charged particles

The work of the Lorentz's self-interaction per unit of time $P^{tot}(t)$ in an isotropic nonabsorbable medium and in vacuum is expressed in Refs 4 and 5 as

$$P^{rad}(t) = \int\limits_{\tau} \left(\vec{j}(\vec{r},t) \frac{1}{c} \frac{\partial \vec{A}^{Dir}(\vec{r},t)}{\partial t} - \rho(\vec{r},t) \frac{\partial \varphi^{Dir}(\vec{r},t)}{\partial t} \right) d\vec{r}. \qquad (1)$$

Here $\vec{j}(\vec{r},t)$ is the current density and $\rho(\vec{r},t)$ is the charge density. The integration is over some volume τ. According to the hypothesis of Dirac [4-7], the scalar $\varphi^{Dir}(\vec{r},t)$ and vector $\vec{A}^{Dir}(\vec{r},t)$ potentials are defined as a half-difference of the retarded and advanced potentials:

$$\varphi^{Dir} = \frac{1}{2}\left(\varphi^{ret} - \varphi^{adv}\right), \quad \vec{A}^{Dir} = \frac{1}{2}\left(\vec{A}^{ret} - \vec{A}^{adv}\right). \qquad (2)$$

After substituting (2) into (1) we obtain the relation for instantaneous radiation power in terms of the spectral distribution

$$P^{rad}(t) = \int_0^\infty d\omega \; W(t,\omega)\,, \tag{3}$$

where

$$W(t,\omega) = \frac{1}{\pi c^2} \int_{-\infty}^{\infty} d\vec{r} \int_{-\infty}^{\infty} d\vec{r'} \int_{-\infty}^{\infty} dt' \omega \mu(\omega) \frac{\sin\left(\frac{n(\omega)}{c}\omega|\vec{r}-\vec{r'}|\right)}{|\vec{r}-\vec{r'}|} \cos\{\omega(t-t')\} \times$$

$$\times \left\{\vec{j}(\vec{r},t)\,\vec{j}\left(\vec{r'},t'\right) - \frac{c^2}{n^2(\omega)}\rho(\vec{r},t)\,\rho\left(\vec{r'},t'\right)\right\}. \tag{4}$$

Here $\mu(\omega)$ is the magnetic permeability, $n(\omega)$ the refraction index, ω the cyclic frequency, and c the velocity of light in vacuum. The time-averaged radiation power of charged particles is determined by the expression

$$\overline{P}^{rad} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt P^{rad}(t) \tag{5}$$

and can be obtained substituting expressions (3) and (4) into (5).

3.**Systems of non-interacting point charged particles**

Let us consider a system of non-interacting point charged particles with charges q_1, q_2,...,q_N and rest masses m_{01}, m_{02},...,m_{0N} moving along an arbitrary trajectory. Then the source functions of the N charged point particles are [5,8]:

$$\vec{j}(\vec{r},t) = \sum_{l=1}^{N} \vec{V}_l(t)\,\rho_l(\vec{r},t)\,, \;\; \rho(\vec{r},t) = \sum_{l=1}^{N} \rho_l(\vec{r},t)\,, \;\; \rho_l(\vec{r},t) = q_l\delta(\vec{r}-\vec{r}_l(t))\,, \tag{6}$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and the velocity of the l^{th} particle, respectively.

Taking into account the source functions (6), and expression (4) we obtain the spectral distribution of the instantaneous radiation power of charged particles moving in isotropic nonabsorbable media ($\epsilon(\omega)$ and $\mu(\omega)$ are real):

$$W(t,\omega)=\frac{1}{\pi c^2}\int\limits_{-\infty}^{\infty}dt'\omega\mu(\omega)\sum_{l,j=1}^{N}q_lq_j\frac{\sin\left(\frac{n(\omega)}{c}\omega|\vec{r}_l(t)-\vec{r}_j(t')|\right)}{|\vec{r}_l(t)-\vec{r}_j(t')|}\times$$
$$\times\cos\{\omega(t-t')\}\left\{\vec{V}_l(t)\vec{V}_j(t')-\frac{c^2}{n^2(\omega)}\right\}. \qquad (7)$$

Substituting relationship (7) into (3), we come to the expression for instantaneous radiation power of a system a charged particles in isotropic nonabsorbable media:

$$P^{rad}(t)=\frac{1}{\pi c^2}\int\limits_{0}^{\infty}d\omega\int\limits_{-\infty}^{\infty}dt'\omega\mu(\omega)\sum_{l,j=1}^{N}q_lq_j\frac{\sin\left(\frac{n(\omega)}{c}\omega|\vec{r}_l(t)-\vec{r}_j(t')|\right)}{|\vec{r}_l(t)-\vec{r}_j(t')|}\times$$
$$\times\cos\{\omega(t-t')\}\left\{\vec{V}_l(t)\vec{V}_j(t')-\frac{c^2}{n^2(\omega)}\right\}. \qquad (8)$$

In the particular case of identical charged particles ($q_l=e$) moving one by one along an arbitrary trajectory, the motion law and velocity of the l^{th} particle of this system are determined by the relations

$$\vec{r}_l(t)=\vec{r}_p(t+\Delta t_l),\quad \vec{V}_l(t)=\vec{V}(t+\Delta t_l). \qquad (9)$$

In this case we obtain the averaged radiation power after substitution of expressions (9) into (8) at taking into account (5):

$$\overline{P}^{rad}=\frac{e^2}{\pi c^2}\lim_{T\to\infty}\frac{1}{2T}\int\limits_{-T}^{T}dt\int\limits_{-\infty}^{\infty}dt'\int\limits_{0}^{\infty}d\omega\mu(\omega)\omega S_N(\omega)\cos\{\omega(t-t')\}\times$$
$$\times\frac{\sin\left\{\frac{n(\omega)}{c}\omega|\vec{r}_p(t)-\vec{r}_p(t')|\right\}}{|\vec{r}_p(t)-\vec{r}_p(t')|}\left\{\vec{V}(t)\vec{V}(t')-\frac{c^2}{n^2(\omega)}\right\}. \qquad (10)$$

Here the coherence factor $S_N(\omega)$ is defined as [3]

$$S_N(\omega) = \sum_{l,j=1}^{N} \cos\left\{\omega\left(\Delta t_l - \Delta t_j\right)\right\}. \tag{11}$$

Therefore, in the case of identical particles the obtained expressions for the averaged radiation power contain the coherence factor that determines a redistribution of the radiation power between harmonics.

4. Fine structure of radiation spectra for electrons moving in a spiral

Peculiarities of the radiation spectra for the electrons moving in a spiral in isotropic nonabsorbable media and in vacuum can be investigated combining analytical and numerical methods. The law of motion and the velocity of the electron are given by the expressions

$$\vec{r}_p(t) = r_0 \cos\omega_0 t\vec{i} + r_0 \sin\omega_0 t\vec{j} + V_{\|} t\vec{k}, \;\; \vec{V}(t) = \frac{d\vec{r}_p(t)}{dt}. \tag{12}$$

Here $r_0 = V_{\perp}\omega_0^{-1}$, $\omega_0 = ceB^{ext}\tilde{E}^{-1}$, $\tilde{E} = c\sqrt{p^2 + m_0^2 c^2}$, $V_{\perp}$ and $V_{\|}$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass, the vector $\vec{B}^{ext}$ of magnetic induction being directed along the Z axis. For the law of motion (12) relationship (10) takes the form

$$\overline{P}^{rad} = \int_0^{\infty} d\omega \; W(\omega), \tag{13}$$

$$W(\omega) = \frac{2e^2}{\pi c^2} \int_0^{\infty} dx \omega \mu(\omega) \frac{\sin\left(\frac{n(\omega)}{c}\omega\eta(x)\right)}{\eta(x)} \times$$
$$\times \cos(\omega x)\left\{V_{\perp}^2 \cos(\omega_0 x) + V_{\|}^2 - \frac{c^2}{n^2(\omega)}\right\}. \tag{14}$$

where

$$\eta(x) = \sqrt{V_{\parallel}^2 x^2 + 4\frac{V_{\perp}^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}. \tag{15}$$

From relations (13) and (14) after some transformations the contribution of separate harmonics to the averaged radiation power can be written as [3, 7, 9]:

$$\overline{P}^{rad} = \frac{e^2}{c^3}\sum_{m=-\infty}^{\infty}\int_0^{\infty} d\omega\mu(\omega)\,n(\omega)\,\omega^2\int_0^{\pi}\sin\theta d\theta\delta\left\{\omega\left(1-\frac{n(\omega)}{c}V_{\parallel}\cos\theta\right)-m\omega_0\right\}\times$$

$$\times\left\{V_{\perp}^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m^{\prime 2}(q)\right]+\left(V_{\parallel}^2-\frac{c^2}{n^2(\omega)}\right)J_m^2(q)\right\}, \tag{16}$$

Here $q = V_{\perp}\dfrac{n(\omega)}{c}\dfrac{\omega}{\omega_0}\sin\theta$, and $J_m(q)$ and $J_m^{'}(q)$ are the Bessel function with integer index and its derivative, respectively.

From relation (16) one can conclude that for velocities $V_{\parallel} < c/n(\omega)$ each harmonics is represented by a set of frequencies, which are determined from the solution of the equation

$$\omega\left(1-\frac{n(\omega)}{c}V_{\parallel}\cos\theta\right)-m\omega_0 = 0. \tag{17}$$

When ϵ and μ are constant, the limits of the m^{th} harmonic are given by the frequencies

$$\omega_m^{min} = m\omega_0\left(1+\frac{n}{c}V_{\parallel}\right)^{-1}, \quad \omega_m^{max} = m\omega_0\left(1-\frac{n}{c}V_{\parallel}\right)^{-1}. \tag{18}$$

Integrating in equation (16) over frequency when ϵ and μ are constant, for velocities $V < c/n$ we obtain the distribution of the radiation power in harmonics [7]. Taking the summation of the series over the Bessel functions and then integrating over θ, we received the total power emitted by the electron [7]

$$P_m^{tot} = \frac{2e^2\mu n}{3c^3}\omega_0^2 V_{\perp}^2\left(1-\frac{n^2V^2}{c^2}\right)^{-2}, \tag{19}$$

where

$$\omega_0 = \omega_{00}\sqrt{1-\frac{V^2}{c^2}},\ \ \omega_{00} = \frac{eB^{ext}}{m_0 c}. \tag{20}$$

The total radiation power P^{tot}_{vac} emitted by the electron moving in a spiral in vacuum is obtained when in relation (19) $\mu = 1$ and $n = 1$. Below we present the results of the numerical calculation of the spectral distribution for the power emitted with the electron moving in a spiral in vacuum and in an ideal dielectric ($\mu = 1$). They are based on relation (14) and were carried out at $B^{ext} = 1Gs$.

Let us consider the case when the velocity vectors in vacuum $\vec{V}_{vac}$ and in dielectric $\vec{V}_m$ are correlated in the form

$$\vec{V}_{vac} = n\vec{V}_m. \tag{21}$$

Then for

$$V^0_m = \frac{c}{\sqrt{n^2+n+1}},\ \ V^0_{vac} = \frac{cn}{\sqrt{n^2+n+1}}, \tag{22}$$

we obtained

$$P^{tot}_m = P^{tot}_{vac}. \tag{23}$$

For the velocities $V^0_{\perp vac} = 0.1\cdot 10^{11}$ cm/s and $V^0_{\| vac} = 0.25\cdot 10^{11}$ cm/s in vacuum, $V^0_{\perp m} = 0.2\cdot 10^{10}$ cm/s and $V^0_{\| m} = 0.5\cdot 10^{10}$ cm/s in a medium with $n = 5$, the spectral distributions of the power emitted by electrons on relative frequency ω/ω_{0j} (curves 1 and 2 in Fig. 1) are identical. The same distributions of the radiation power with frequency ω are shown in Fig.2.

In vacuum ($\mu = 1$, $n = 1$) the total radiation power $P^{tot}_{vac} = 0.913\cdot 10^{-15}$ erg/s, calculated according to relation (19) is in good agreement to $P^{int}_{vac} = 0.91\cdot 10^{-15}$ erg/s, which was obtained after integration over ω in relation (13) and taking into account (14). In the medium the corresponding radiation powers are: $P^{tot}_m = 0.914\cdot 10^{-15}$ erg/s and $P^{int}_m = 0.915\cdot 10^{-15}$ erg/s.

For the refraction index $n = 5.6$ at the velocity $V_{\perp m} = 0.2\cdot 10^{10}$ cm/s and $V_{\| m} = 0.5\cdot 10^{10}$ cm/s the conditions for existence of synchrotron-Cherenkov radiation are satisfied. As follows from curve 3 in Fig. 3, at $V > c/n$ the synchrotron-Cherenkov radiation is the only process in the medium [3, 7- 8, 10]. At rectilinear motion the Cherenkov radiation power is given as

$$P^{tot}_{ch} = \frac{e^2}{2c^2}V\omega^2_{max}\left(1-\frac{c^2}{V^2n^2}\right). \tag{24}$$

In the case when $V_\perp << V$ the synchrotron-Cherenkov radiation spectrum (curve 4 in Fig.3) differs very little from the Cherenkov radiation spectrum of the rectilinear motion in an ideal dielectric.

At low velocities, $V_{\perp vac} = 0.2 \cdot 10^{10}$ cm/s and $V_{||vac} = 0.5 \cdot 10^{10}$ cm/s the radiation takes place at the basic frequency (the first harmonic), which due to the Doppler effect influence broadens into a band (curve 5 in Fig.4). Its limits are determined by relation (18). In the case when $V_{\perp vac} = 0.6 \cdot 10^{10}$ cm/s and $V_{||vac} = 0.15 \cdot 10^{11}$ cm/s the radiation takes place at the first and second harmonics (curve 6 in Fig.4). As one can see from fig.4, at low electron velocity when the electron is moving in circle, the radiation occurs preferably in the both directions perpendicular to the plane (the XY plane).

The radiation power of electrons calculated integrating over ω in (13) the spectral distribution (14) are in good agreement to those calculated using the exact relation (19). It justifies the use of numerical calculations. The results obtained here are in agreement with the data of Refs. [11,12].

Conclusions

The integral expressions for spectral distributions of the instantaneous and time-averaged radiation powers of non-interacting system of point charged particles moving on an arbitrary trajectories in isotropic nonabsorbable media and in vacuum are obtained.

We have investigated the fine structure of radiation spectrum emitted with electrons moving in a spiral in a constant magnetic field in vacuum and in ideal dielectric. The obtained spectral distributions of synchrotron, Cherenkov, and synchrotron-Cherenkov radiations can be used to find new sources of electromagnetic energy.

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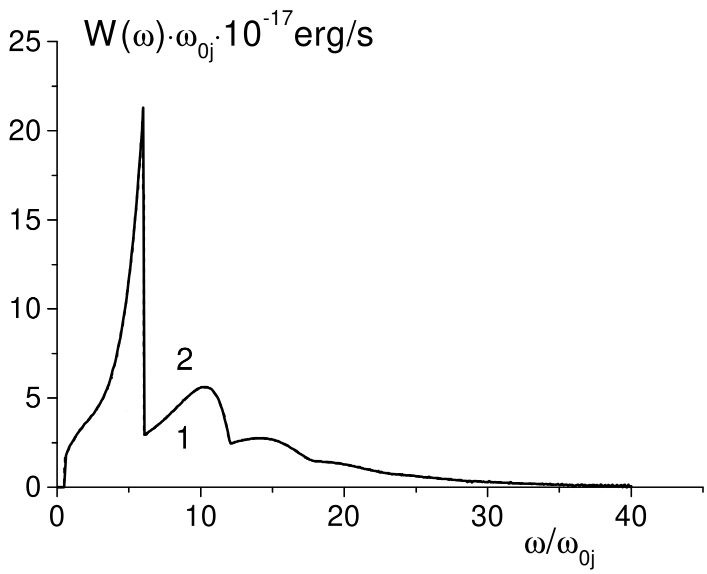

Figure 1: Spectral distribution of synchrotron radiation power with relative frequency

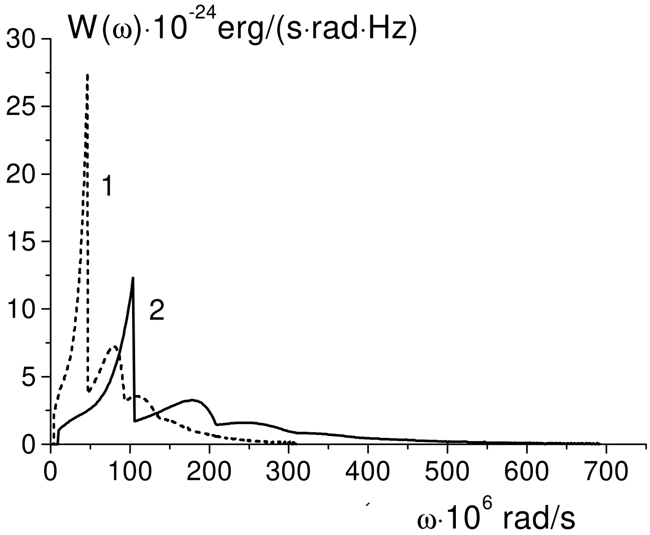

Figure 2: Spectral distribution of synchrotron radiation power with frequency. Curve 1: $n = 1$, $V^0_{\perp vac} = 0.1 \cdot 10^{11}$ cm/s, $V^0_{\| vac} = 0.25 \cdot 10^{11}$ cm/s, $\omega_{01} = 0.733 \cdot 10^7$ rad/s, $r_{01} = 1293.1$ cm, $P^{tot}_{vac} = 0.913 \cdot 10^{-15}$ erg/s, $P^{int}_{vac} = 0.91 \cdot 10^{-15}$ erg/s. Curve 2: $n = 5$, $V^0_{\perp m} = 0.2 \cdot 10^{10}$ cm/s, $V^0_{\| m} = 0.5 \cdot 10^{10}$ cm/s, $\omega_{02} = 0.173 \cdot 10^8$ rad/s, $r_{02} = 115.6$ cm, $P^{tot}_{m} = 0.914 \cdot 10^{-15}$ erg/s, $P^{int}_{m} = 0.915 \cdot 10^{-15}$ erg/s.

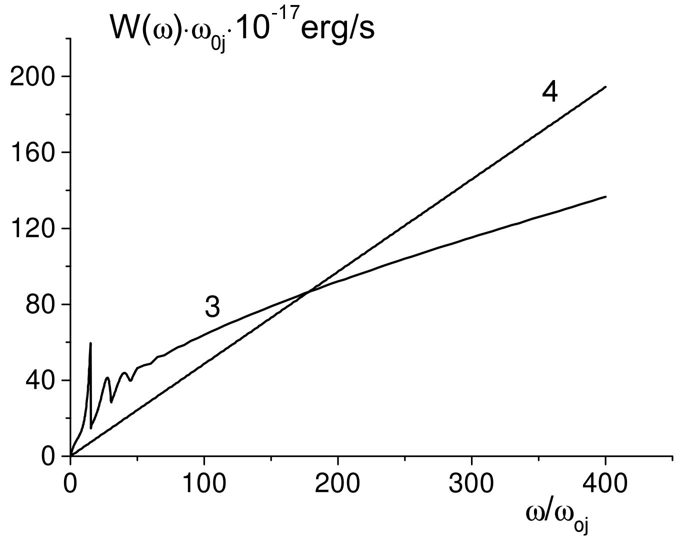

Figure 3: Spectral distribution of synchroton-Cherenkov radiation power with relative frequency. Curve 3: $n = 5.6$, $V_{\perp m} = 0.2 \cdot 10^{10}$ cm/s, $V_{\|m} = 0.5 \cdot 10^{10}$ cm/s, $\omega_{03} = 0.173 \cdot 10^{8}$ rad/s, $r_{03} = 115.6$ cm, $P_m^{int} = 0.3555 \cdot 10^{-12}$ erg/s. Curve 4: $n = 5.6$, $V_{\perp m} = 0.1 \cdot 10^{8}$ cm/s, $V_{\|m} = 0.5385 \cdot 10^{10}$ cm/s, $\omega_{04} = 0.173 \cdot 10^{8}$ rad/s, $r_{04} = 0.6$ cm, $P_{ch}^{tot} = 0.3889 \cdot 10^{-12}$ erg/s, $P_m^{int} = 0.3899 \cdot 10^{-12}$ erg/s.

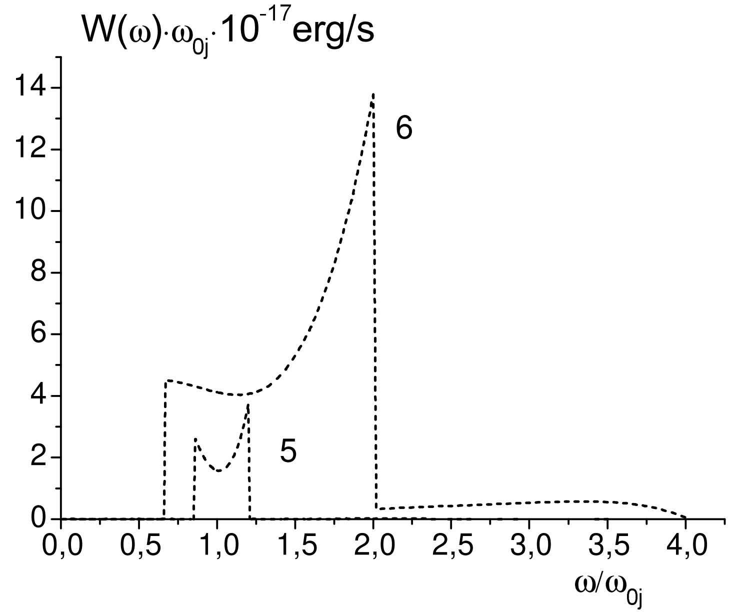

Figure 4: Spectral distribution of synchrotron radiation power with relative frequency. Curve 5: $n = 1$, $V_{\perp vac} = 0.2 \cdot 10^{10}$ cm/s, $V_{||vac} = 0.5 \cdot 10^{10}$ cm/s, $\omega_{05} = 0.173 \cdot 10^8$ rad/s, $r_{05} = 155.6$ cm, $P_{vac}^{tot} = 0.7298 \cdot 10^{-17}$ erg/s, $P_{vac}^{int} = 0.745 \cdot 10^{-17}$ erg/s. Curve 6: $n = 1$, $V_{\perp vac} = 0.6 \cdot 10^{10}$ cm/s, $V_{||vac} = 0.15 \cdot 10^{11}$ cm/s, $\omega_{06} = 0.148 \cdot 10^8$ rad/s, $r_{06} = 405.0$ cm, $P_{vac}^{tot} = 0.8958 \cdot 10^{-16}$ erg/s, $P_{vac}^{int} = 0.8935 \cdot 10^{-16}$ erg/s.