Neutrino masses from double-beta decay

S. Stoica[a] and H.V. Klapdor-Kleingrothaus[b]

a) *Theoretical Physics Department, National Institute of Physics and Nuclear Engineering, P.O. Box MG-6, 76900-Bucharest, Romania*

b) *Max-Planck Institut für Kernphysik, W-6900 Heidelberg, Germany*

November 28, 2002

Abstract. We make a systematic study of the neutrinoless double-beta decay matrix elements (ME) for several nuclei of experimental interest. The calculations are performed with the second quasi random phase approximation (SQRPA) methods. A better stability against the change of the s.p. basis used and a good fulfillment of the Ikeda Sum Rule allow to reduce the uncertaintes in the values of the neutrinoless ME predicted by the QRPA-based methods to about 50% from their calculated values. Further, using the most recent experimental limits for the neutrinoless half-lives, we derive new upper limits for the neutrino masses. These are in agreement with the recent claim of experimental evidence for neutrinoless double-beta decay.

1 Introduction

The extensions of the proton-neutron random phase approximation method (pnQRPA) beyond the quasiboson approximation (QBA) have received much attention in the last decade in the context of nuclear structure calculations for the double-beta decay. The pnQRPA which has been widely used in such calculations (for recent reviews see Refs. [1]-[5]) has succeeded to reproduce the experimentally observed suppression of the two-neutrino double beta ($2\nu\beta\beta$) decay matrix elements. By including the particle-particle residual interaction between nucleons, which is attractive in the 1^+ pn channel, besides the particle-hole one which is repulsive, one can get arbitrarily small matrix elemetns from the destructive interference between them [6]-[11]. However, the price paid is a strong sensitivity of the matrix elements to the increase of the strength of these residual interaction in the 1^+ channel and leads to the problem of fixing this parameter. Since the Shell Model- (SM) based methods, despite of their recent progress [23] are not yet able to treat medium heavy off-shell nuclei, which are the most part of the $\beta\beta$ decay emitters, one further relies on QRPA-based methods. Thus, various refinements of the original pnQRPA have been advanced but the most attention have received the approaches which go beyond the QBA.

The first method which includes higher-order corrections to the pnQRPA was developed in Refs. [14] and applied to the evaluation of the $2\nu\beta\beta$ decay ME of ^{82}Se. In this approach the pnQRPA phonon operator and the transition $\beta^{\pm}$ operators have been expressed as boson expansions of appropriate pair operators and there were kept the next order terms from these series beyond the quasi boson approximation. Then, this method, but keeping only the two-boson contributions to the wave function, was called SQRPA and has been employed, with some numerical improvements, for similar calculations for other isotopes as well as for transitions to excited states [15]-[16].

An alternative approach, called pnRQRPA, was developed in ref. [18] and then used extensively for both 2ν- and $0\nu-\beta\beta$ decay modes, for transitions to g.s. and excited states and for different nuclei [4], [19]-[21]. Within this method one tries a partial restoration of the Pauli exclusion principle which is discarded in the pnQRPA. This is done by taking into account the next terms in the commutator expression of the like-nucleon operators involved in the derivation of the pnQRPA equations. The commutator is replaced by its expectation value in the RPA (correlated) g.s. and this leads to a renormalization of the relevant operators and of the forward- and backward-going QRPA amplitudes as well. The pnRQRPA method has been further refined by the inclusion of pn pairing interactions besides the proton-proton and neutron-neutron ones [20] and this method, called full-RQRPA, was widely used $\beta\beta$ decay calculations [20], [22].

Both methods, by the inclusion of higher-order corrections to the QBA, display a common feature, namely, the ME become more stable against g_{pp} and the point where the pnQRPA solutions become imaginary is shifted towards the region of unphysical values of this parameter.

However, within the pnRQRPA the Ikeda Sum Rule (ISR) is not conserved by a significant amount. Another drawback of this method would be a rather large dependence of the calculated ME upon the size of the s.p. basis, as it was shown in some previous papers [20]-[21], [33]. These shortcomings have not been found in a significant proportion in the second-QRPA (SQRPA) method [16], [33] for $2\nu\beta\beta$ decay calculations. Since calculations of neutrinoless ($0\nu\beta\beta$) ME with SQRPA have not been done until present, it of interest to see that features also hold for this decay mode.

In this article we make a computation of the nuclear $0\nu\beta\beta$ decay ME with the SQRPA method for the experimentally interesting nuclei ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, $^{128,130}Te$, and ^{136}Xe. The main goal is to reduce the uncertainties found in the QRPA-based methods in the prediction of these matrix elements, which are related to the change of the s.p. basis and the conservation of the ISR. For that we used two different s.p. basis and fixed the parameters needed in the numerical procedure in the same manner as it was done in our work [33], where similar calculations with the other QRPA-based methods are also performed. By comparing the present calculations with others found in the literature we estimate the predictive power of the QRPA methods concerning the ME for the neutrinoless decay mode. Then, using the most recent $0\nu\beta\beta$ half-lives reported in the literature, we deduce new limits for the neutrino mass parameter.

2 Formalism

In the QRPA-based methods the excitation operator can be defined in the following general form:

$$\Gamma^{m+}_{JM^{\pi}} = \sum_{k,l,\mu\leq\mu'} \left[X^{m}_{\mu\mu'}(k,l,J^{\pi}) A^{\dagger}_{\mu\mu'}(k,l,J,M) + Y^{m}_{\mu\mu'}(k,l,J^{\pi}) \tilde{A}_{\mu\mu'}(k,l,J,M) \right] \quad (1)$$

where $k \leq l$ if $\mu = \mu'$, X and Y are the forward- and backward-going QRPA amplitudes and $A, A^{\dagger}$ are the bifermion quasiparticle operators coupled to angular momentum J and projection M:

$$A^{\dagger}_{\mu\mu'}(k,l,J,M) = \mathcal{N}(k\mu, l\mu') \sum_{m_k,m_l} C^{JM}_{j_k m_k j_l m_l} a^{\dagger}_{\mu k m_k} a^{\dagger}_{\mu' l m_l}$$

$$\tilde{A}_{\mu\mu'}(k,l,J,M) = (-)^{J-M} A_{\mu\mu'}(k,l,J,-M) \quad (2)$$

$\mathcal{N}$ is a normalization constant, which differs from unity only in case when both quasi-particles are in the same shell [19], $\mu, \mu' = 1,2$ and $1 \equiv$protons, $2 \equiv$neutrons. Using for instance the equation of motion method one can derive the pnQRPA equations which may be written in the matrix representation as follows:

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{pmatrix}_{J^{\pi}} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} = \Omega^m_{J^{\pi}} \begin{pmatrix} \mathcal{U} & 0 \\ 0 & -\mathcal{U} \end{pmatrix}_{J^{\pi}} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} \quad (3)$$

with

$$\mathcal{A}_J(\mu k, \nu l; \mu' k', \nu' l') = \langle 0^+_{RPA} | \left[A_{\mu\nu}(k,l,J,M), [\hat{H}, A^{\dagger}_{\mu'\nu'}(k',l',J,M)] \right] | 0^+_{RPA} \rangle$$

$$\mathcal{B}_J(\mu k, \nu l; \mu' k'; \nu' l') = \langle 0^+_{RPA} | \left[A_{\mu\nu}(k,l,J,M), [\tilde{A}_{\mu'\nu'}(k',l',J,M), \hat{H}] \right] | 0^+_{RPA} \rangle \quad (4)$$

$$\mathcal{U} = \langle 0^+_{RPA}| \left[A_{\mu\nu}(k,l,J,M), [A^\dagger_{\mu'\nu'}(k',l',J,M)]\right] |0^+_{RPA}\rangle \tag{5}$$

Here the $\Omega^m_{J^\pi}$ are the QRPA excitation energies for the mode J^π.

The simplest way to calculate the $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{U}$ matrices is to adopt the so-called QBA, as is done within the pnQRPA method, i.e. the quasiparticle operators $A, A^\dagger$ are assumed to behave like bosons and satisfy thus exactly the boson commutation relations:

$$\left[A_{\mu\nu}(k,l,J,M), A^\dagger_{\mu'\nu'}(k',l',J,M)\right] =$$
$$\mathcal{N}(k\mu,l\nu)\mathcal{N}(k'\mu',l'\nu')\left(\delta_{\mu\mu'}\delta_{\nu\nu'}\delta_{kk'}\delta_{ll'} - \delta_{\mu\nu'}\delta_{\nu\mu'}\delta_{lk'}\delta_{kl'}(-)^{j_k+j_l-J}\right) \tag{6}$$

In the SQRPA method the higher-order corrections to the QRPA are included by expanding the $A^\dagger$, A together with the quasiparticle-density dipole operators into a series of boson operators, from which one retains the next terms beyond the QBA [14]:

$$A^\dagger_{1\mu}(pn) = \sum_k \left(A^{(1,0)}_{k_1}\Gamma^+_{1\mu}(k) + A^{(0,1)}_{k_1}\tilde{\Gamma}^+_{1\mu}(k)\right) \tag{7}$$

$$B^\dagger_{1\mu}(pn) = \sum_{k_1k_2}\left(B^{(2,0)}_{k_1k_2}(pn)[\Gamma^\dagger_1(k_1)\Gamma^\dagger_2(k_2)]_{1\mu} + B^{(0,2)}_{k_1k_2}(pn)[\Gamma_1(k_1)\Gamma_2(k_2)]_{1\mu}\right) \tag{8}$$

where

$$B^\dagger_{1\mu}(pn) = \sum_{m_k,m_l} C^{JM}_{j_pm_pj_nm_n} a^\dagger_{j_pm_p} a_{j_nm_n}$$

$$\tilde{B}_{1\mu}(pn) = (-)^{J-M}B_{1\mu}(pn) \tag{9}$$

The boson expansion coefficients $A^{(1,0)}$, $A^{(1,0)}$, $B^{(2,0)}$, $B^{(0,2)}$ are determined so that the equations (7)-(8) are also valid for the corresponding ME in the boson basis. Further, in the quasiparticle representation, the transition $\beta^\pm$ operators can also be expressed in terms of the dipole operators $A_{1\mu}$ and $B_{1\mu}$:

$$\beta^-_\mu(k) = \theta_k A^\dagger_{1\mu}(k) + \bar{\theta}_k\tilde{A}_{1\mu} + \eta_k B^\dagger_{1\mu}(k) + \bar{\eta}_k\tilde{B}_{1\mu}$$
$$\beta^+_\mu(k) = -\left(\bar{\theta}_k A^\dagger_{1\mu}(k) + \theta_k\tilde{A}_{1\mu} + \bar{\eta}_k B^\dagger_{1\mu}(k) + \eta_k\tilde{B}_{1\mu}\right) \tag{10}$$

where

$$\theta_k = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle U_pV_n;\ \bar{\theta}_k = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle U_nV_p;\ \hat{j} = \sqrt{2j+1}$$

$$\eta_k = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle U_pU_n;\ \bar{\eta}_k = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle V_pV_N \tag{11}$$

For consistency, by using (7)-(8) their expressions are also obtained in the same order beyond quasi boson approximation. Their complete expressions can be found in Ref. [14]. It is important to stress that these expressions contain, besides the one-boson terms present in the QBA, higher-order contributions which are proportional to products of two-boson operators. Thus, in the SQRPA method, the higher-order corrections to the pnQRPA are consistently introduced both in the wave functions (through the phonon operators), and in the expressions of the $\beta^{\pm}$ operators.

For the $0\nu\beta\beta$ decay we assume in this paper only the mass mechanism and so the half-life can be written in the factorized form as follows(see e.g. [2]:

$$\left[T_{1/2}^{0\nu}\right]^{-1} = C_{mm}\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2 \tag{12}$$

where $\langle m_\nu\rangle$ is the effective neutrino mass and

$$C_{mm} = F_1^{0\nu}\left(M_{GT}^{0\nu} - \left(\frac{g_v}{g_A}\right)^2 M_F^{0\nu}\right)^2 \tag{13}$$

$F_1^{0\nu}$ is the phase-space integral and $M_{GT}^{0\nu}$ and $M_F^{0\nu}$ are Gamow-Teller and Fermi matrix elements.

We also mention that in all models the Ikeda sum rule must be fulfilled, i.e.:

$$S_- - S_+ = \Sigma_m|\langle 0^+_{g.s.}||\beta^-_m||1^+_m\rangle|^2 - |\langle 1^+_{gs}||\beta^+_m||0^+_{g.s.}\rangle|^2 = \Sigma_m(-)^m\langle 0^+_{g.s.}|[\beta^+_m,\beta^-_m]|0_{g.s.}\rangle =$$

$$= 3(N-Z) \tag{14}$$

3 Numerical results

In the calculations of the neutrinoless ME for ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te and ^{136}Xe, for the Hilbert space used to generate the s.p. basis we made two choices. For the nuclei with $A \leq 100$ we included: i) the full $(3-4)\bar{h}\omega$ oscillator shells and ii) the full $(2-4)\bar{h}\omega$ oscillator shells. For the nuclei with $A > 100$ the two s.p. basis include: i) the full $(3-5)\bar{h}\omega$ oscillator shells and ii) the full $(2-5)\bar{h}\omega$ oscillator shells. From here on we will call (s) the small basis (i) and (l) the large basis (ii) and these indices are used in tables and figures to distinguish between calculations performed with the two basis.

The s.p. energies were obtained by solving the Schrӧdinger equation with a Coulomb-corrected Wood-Saxon potential. The λ-pole nucleon-nucleon residual interactions were taken as Brueckner G-matrix derived from the Bonn-A one-pion-exchange potential.

The quasiparticle energies and the BCS occupation amplitudes were calculated by solving the HFB equations with proton-proton (pp) and neutron-neutron (nn) pairing correlations. The calculations were performed separately for the initial and final nuclei participating in the $\beta\beta$ decay and for the two basis sets. Also, we included in the model space the states with all multipolarities J^{π}. The renormalization constants were chosen as follows: $g_{pp} = 1.0$ for all the multipolarities, except the 1^+ channel for which it was left as a free parameter, and $g_{ph} = 1.0$ for all the multipolarities except the 2^+ channel where it was fixed to 0.8, since for larger values the p-h interaction in this channel is too strong producing the collapse of the RPA procedure. For deriving the neutrino mass, the g_{pp} parameter was fixed from our two-neutrino calculations performed with the SQRPA method [33] and according to the most recent corresponding half-lives found in the literature. We study the dependence of the neutrinoless matrix elements upon the g_{pp} parameter, for the two choices of the s.p. basis, for the nuclei investigated. One observes that the matrix elements display a weak dependence on the size of the s.p. basis used and this is valid for all the nuclei. In the vicinity of the fixed value of g_{pp} the differences between calculations, performed with the two different basis, for the same nucleus, are less than 30%. Thus, one may conclude that the prediction of the ME for the neutrinoless decay mode given by the SQRPA, related to the use of different s.p. basis, is reasonably good. Distinctively, in [20] a significant dependence (up to a factor 3) was found for similar calculations performed with pnQRPA, RQRPA and full-RQRPA. Close values between the ME calculated with the full-RQRPA method using different basis sets were obtained only when the basis include more than three full oscillator shells, which might be in disagreement with one of the basic approximation made within the RQRPA method. Indeed, when one enlarges the starting s.p. basis too much (i.e. if one includes more than 2-3 full oscillator shells), one may expect that the overlap matrix $\mathcal{U}$, appearing in the QRPA equation (3) can no more be assumed, with a good approximation, to be diagonal, and so the RQRPA equations can no more be obtained in a QRPA-type closed form.

Then, we checked the Ikeda sum rule within SQRPA and found that it is conserved with good accuracy. Indeed, the deviations from the exact fulfillment, for all the nuclei, are within a few percent. By contrast, we found deviations up to 21% within RQRPA methods [33]. An explanation of the good degree of conservation of the ISR within the SQRPA would be the presence in the expressions of the transition operators of additional terms as it was shown before. These higher-order terms, corroborate with the improved expression of the SQRPA wave function, in the calculation of the left hand side of Eq. (14) and give positive contributions to the fulfillment of the ISR.

These two features, e.g. a more stable behavior of the results against the change of the s.p. basis and the fulfillment with good approximation of the ISR give more confidence in the values of the ME obtained with the SQRPA method.

By comparing the results obtained in this paper with similar results obtained with other methods, one can estimate more precisely the accuracy in the prediction of the neutrinoless matrix elements with the QRPA-based methods.

For that in table 1 we give the values of the neutrinoless ME calculated with different methods. One observes that the discrepancies between the values obtained with recent pnQRPA, RQRPA and SQRPA calculations, are generally within a range of 50%. There are however two exceptions e.g. the values for ^{100}Mo, calculated with pnQRPA in Ref. [12], and for ^{76}Ge, calculated with the full-RQRPA in Ref. [20] which are smaller by factors of 2-3 as compared with other calculations. One also observes that the values obtained with the SM are smaller by factors up to two than similar QRPA calculations. This might be understood by the poor conservation of the ISR within this method(about 50% from the beta strength is lost). Thus, one can estimate that with the present calculations the uncertainty in the prediction of the ME for the neutrinoless decay mode, by the QRPA-based methods may be reduced to about 50%, if we relate the discrepancies to the non-conservation of the ISR and to the change of the s.p. basis. Since the ME and the neutrino mass parameter enter the half-life formulae at the same power, the same amount of uncertainty is also expected for the neutrino mass predictions.

It should be mentioned that uncertainties in the predictions of the neutrinoless ME might also come from other sources. Recently, it was shown that the inclusion of nucleon currents in the neutrino mass mechanism can reduce the ME by factors of 25-30% [22]. Also, a consistent treatment of the BCS and RQRPA vacua, might give additional corrections [21].

4 Conclusions

Concluding, we have computed the $0\nu\beta\beta$ decay matrix elements, for the first time, with the SQRPA method for the experimentally interesting nuclei: ^{76}Ge, ^{82}Se, ^{96}Zr,^{100}Mo,^{116}Cd,^{128}Te, ^{130}Te and ^{136}Xe using two different choices of the s.p. basis. We found a weak dependence of the matrix elements on the s.p. basis used. Also, the ISR was checked and found that it is fulfilled with good accuracy within the SQRPA method. Both these features give us more confidence in the results. By comparing the present results with the results obtained with other QRPA-based methods and with the SM, one can estimate more reliably the predictive power of the QRPA-based methods concerning the values of the neutrinoless matrix elements. The uncertainty related to the non-conservation of the ISR and to the change of the s.p. basis can be settled within 50 %. The same uncertainty then holds for the neutrino mass predictions. Finally, using the most recent experimental limits for the neutrinoless half-lives found in literature we deduce new limits for the neutrino mass parameter for each case.

References

[1] W.C. Haxton and G.J. Stephenson, Prog. Part. Nucl. Phys. **12** (1984) 409.

[2] K. Muto and H.V. Klapdor-Kleingrothaus, in Neutrinos, ed. H.V. Klapdor, Heidelberg, Springer, 1988, 183.

[3] M. K. Moe and P. Vogel, Annu. Rev. Nucl. Part. Sci. **44** (1984) 247.

[4] J. Suhonen and O. Civitarese, Phys. Rep. **300** (1998) 123.

[5] A. Faessler and F. Simcovic, hep-ph/**9901215**.

[6] P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. **57** (1986) 3148.

[7] K. Grotz and H.V. Klapdor, Nucl. Phys. **A 460** (1986) 395.

[8] T. Tomoda and A. Faessler, Phys. Lett. **B 199** (1987) 475.

[9] K. Muto, E. Bender and H.V. Klapdor, Z. Phys. **A 334** (1989) 177; Z. Phys. **A 334** (1989) 187.

[10] J. Suhonen, A. Faessler, T. Taigel and T. Tomoda, Phys. Lett. **B 202** (1988) 174.

[11] A. Staudt, T.T.S. Kuo and H.V. Klapdor-Kleingrothaus, Phys. Lett. **B 242** (1990) 17.

[12] A. Staudt, K. Muto and H.V. Klapdor-Kleingrothaus, Europhys. Lett. **13** (1990) 31.

[13] T. Tomoda, Rep. Prog. Phys. **54** (1991) 53.

[14] A. A. Raduta, A. Faessler, S. Stoica and W. A. Kaminski,
Phys. Lett. **B 254** (1991) 7; A.A. Raduta, A. Faessler and S. Stoica, Nucl. Phys. **A 534** (1991) 149.

[15] S. Stoica and W.A. Kaminski, Phys. Rev.**C 47** (1993) 867; S. Stoica, Phys. Rev. **C 49** (1994) 787.

[16] S. Stoica, Phys. Lett. **B 350** (1995) 152;
S. Stoica and I. Mihut, Nucl. Phys. **A 602** (1996)

[17] A.A. Raduta, D.S. Delion and A. Faessler, Phys. Rev. **C 51** (1995) 3008.

[18] J. Toivanen and J. Suhonen, Phys. Rev. Lett. **75** (1995) 410; Phys. Rev. **C 55** (1997) 2314.

[19] J. Schwieger, F. Simcovic and A. Faessler, Nucl. Phys. **A 600** (1996) 179.

[20] F. Simcovic, J. Schwieger, M. Veselsky, G. Pantis and A. Faessler, Phys. Lett. **393** (1997) 267;
F. Simcovic, J. Schwieger, G. Pantis and A. Faessler, nucl-th/**9708030**.

[21] A. Bobyk, W.A. Kaminski and P. Zareba, Eur. Phys. J. **A 5** (1999) 385; Nucl. Phys. **A669** (2000) ...

[22] G. Pantis and F. Simcovic, Nucl. Phys. **A 663&664** (2000) 825c.

[23] E. Caurier, F. Nowacki, A. Poves and J. Retamosa, Phys. Rev. Lett. **77** (1996) 1954.

[24] A.A. Raduta and J. Suhonen, Phys. Rev. **C 53** (1996) 176; J. Phys. **G 22** (1996) 123.

[25] T. Bernatovicz et al., Phys. Rev. Lett. **69** 2341; Phys. Rev. **C 47** (1993) 806.

[26] D. Dassie et al., Phys. Rev. **D 51** (1995) 2090.

[27] R. Luescher, Nucl. Phys. B (Proc. Suppl.) **66** (1998) 195.

[28] A. Alessandrello et al., Nucl. Phys. **B 35** (Proc. Suppl.) (1994) 366; Phys. Lett. **B 433** (1998) 156.

[29] A.S. Barabash, Nucl. Phys. **A 629** 1998 517c.

[30] R. Arnold et al., Nucl. Phys. **A 636** (1998) 209; ibidem **A 658** (1999) 299.

[31] H.V. Klapdor-Kleingrothaus, "60 Years of Double-Beta Decay - From Nuclear Physics to Beyond Standard Model Particle Physics", World Scientific, Singapore, 2000; A. Dietz, Diploma thesis, MPI-Heidelberg.

[32] F. A. Danevich et al., nucl-ex/0003001, 1 March 2000.

[33] S. Stoica and H. V. Klapdor-Kleingrothaus, Eur. J. (2001), in press; and submitted to Nucl. Phys. **A**.

$M^{0\nu}$	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe
SQ	3.21(l)	3.54(l)	2.12(l)	4.23(l)	2.29(l)	2.85 (l)	2.42(l)	0.98(l)
	3.78(s)	4.13(s)	2.70(s)	4.51(s)	2.67 (s)	3.38(s)	2.53(s)	1.03(s)
FRQ [20]	1.86			4.22	2.47	3.28	3.19	0.96
FRQ [33]	2.43(l)	2.63(l)	2.42(l)	4.11(l)	2.35(l)	2.88 (l)	2.61(l)	0.89(l)
	3.73(s)	4.15(s)	2.99(s)	4.35(s)	2.62 (s)	3.75(s)	3.49(s)	0.99(s)
Q [12]	4.25	3.99	2.94	1.37	3.38	4.49	3.61	1.65
Q [13]	3.36	3.06		3.04		3.86	3.38	1.12
SM [23]	1.57	1.97						0.65

Table 1: The neutrinoless matrix elements calculated with SQRPA (SQ) (present work) and with other methods: pnQRPA (Q), pnRQRPA (RQ), full-RQRPA (FRQ) and Shell Model (SM). In parenthesis are indicated the references where the calculations are taken from. The indices (l) and (s) refer to the calculations performed with a large or small s.p. basis, respectively