D. DECAY PROCESSES

D.01. Decay modes

D.02. Decay law

D.03. Alpha decay & cluster emission

D.04. Beta decay

D.05. Gamma decay

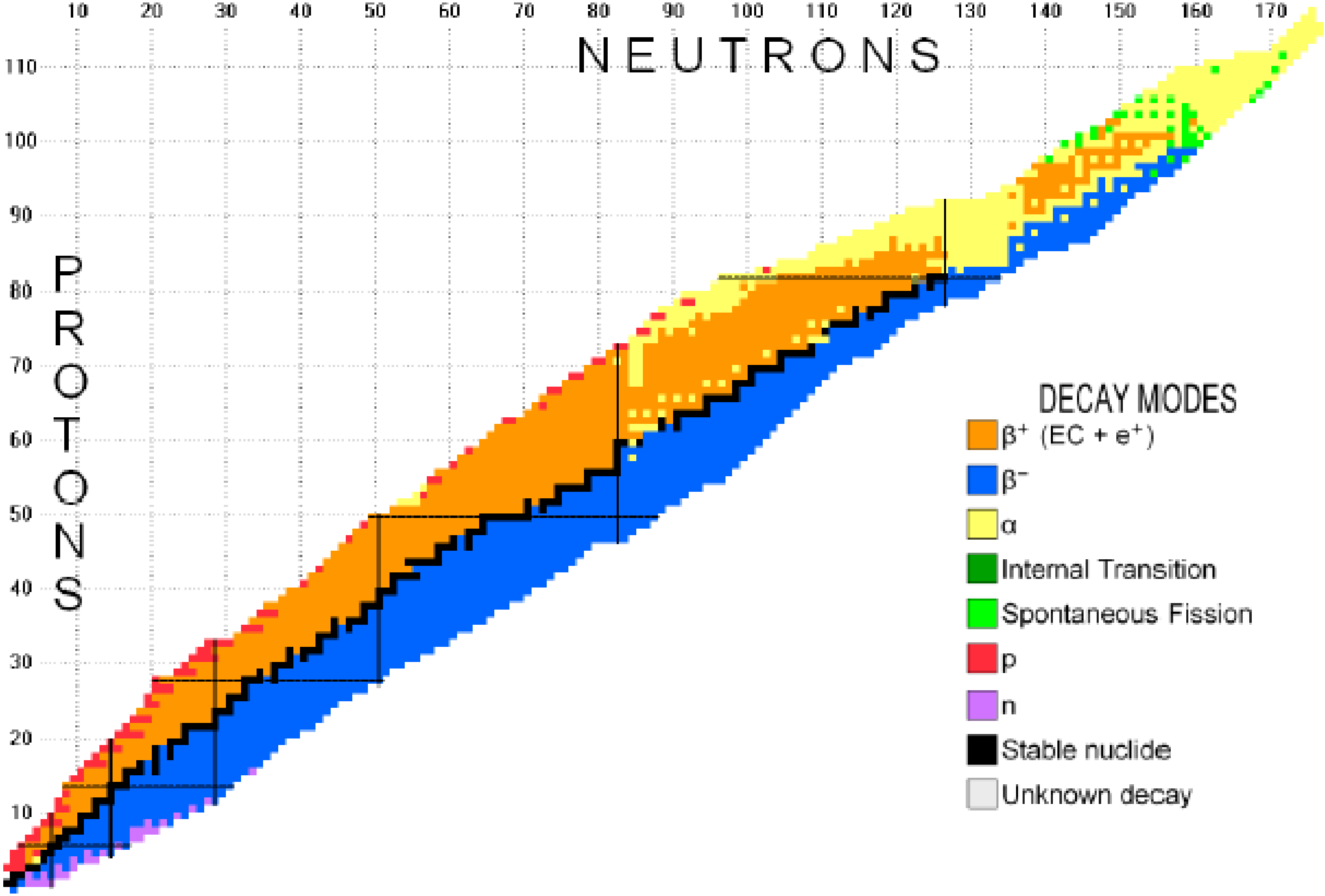
D.06. Fission & fusion

D.01. Decay modes

α decay: emission of ^{4}He
β decay: emission of e^{-} or e^{+}
γ decay: emission of high energy photons

p decay: particle (proton/neutron) emission
c decay: emission of heavy clusters (C,O,Ne,Mg)
sf: spontaneous fission

Nuclear chart of decay modes



D.02. Decay law

$$dN \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

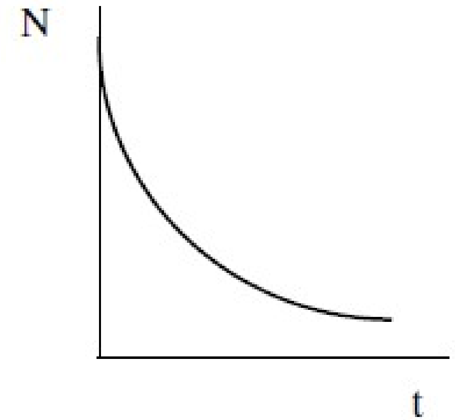
$$N(t) = N_0 e^{-\lambda t}$$

λ = decay constant = probability of a nucleus decaying per second

Half-life = time for half the nuclei to decay

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Lifetime (average) $\tau = \frac{1}{\lambda}$



Decay width is defined by:

$$\Gamma = \hbar\lambda$$

Wave function describing a decaying state

Decay law

$$\left|\Psi_{ext}(R,t)\right|^2 \rightarrow N(t) = N_0 e^{-\lambda t}$$

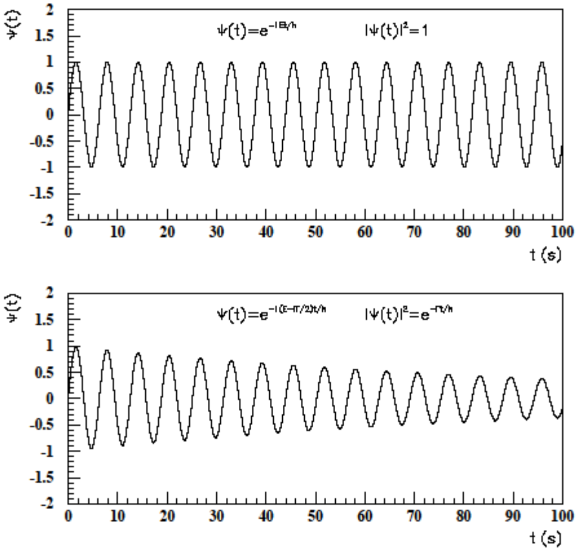
implies an external wave function

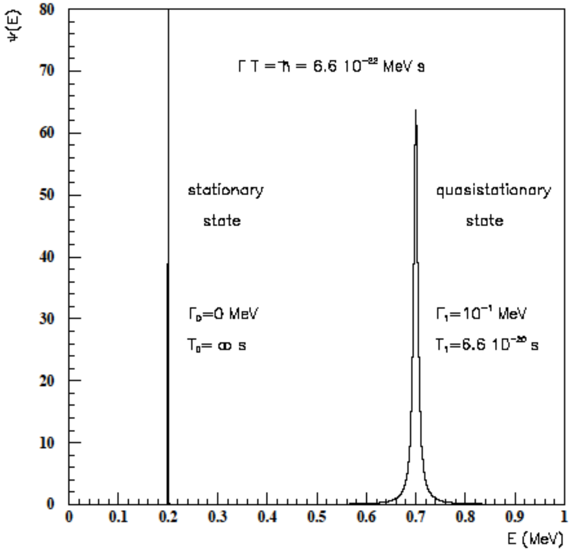
$$\Psi_{ext}(R,t) = \Psi_{ext}(R) e^{-i(E/\hbar)t}$$

with complex energy

$$E = E_0 - i\frac{\Gamma}{2}$$

The width of the energy distribution given by the Fourier transform of a decaying time-dependent signal is proportional to the decay width





Decay rate (activity) for the emission process

P(arent) → D(aughter)+C(luster)

$$R = \left| \frac{dN}{dt} \right| = \lambda N_0 e^{-\lambda t} = \lambda N(t)$$

Units

Becquerel (B) = 1 decay / second

Curie (C) = 3.7 10^{10} B (1 gram of Radium)

D.03. Alpha decay

α-particle is a ^{4}He nucleus (2p & 2n)

- Z ↓ by 2
- N ↓ by 2
- A ↓ by 4

$$^{A}_{Z}X_{N} \rightarrow ^{A-4}_{Z-2}Y_{N-2} + \alpha$$

"parent" "daughter"

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + \alpha$$

$$t_{1/2} = 4.47 \times 10^{9}\ yrs$$

Disintegration energy

$$Q = \left(M_P - M_D - m_\alpha\right)c^2$$

$$M_U = 238.050784\, u$$

$$M_{Th} = 234.043593\, u$$

$$m_\alpha = 4.002602\, u$$

$$M_U - M_{Th} - m_\alpha = 0.004589\, u$$

$$Q = \Delta m \cdot c^2 = 0.004589 \times 931.502$$

$$= 4.275 \quad MeV$$

Theory of the α-decay
(and emission of heavier charged clusters)

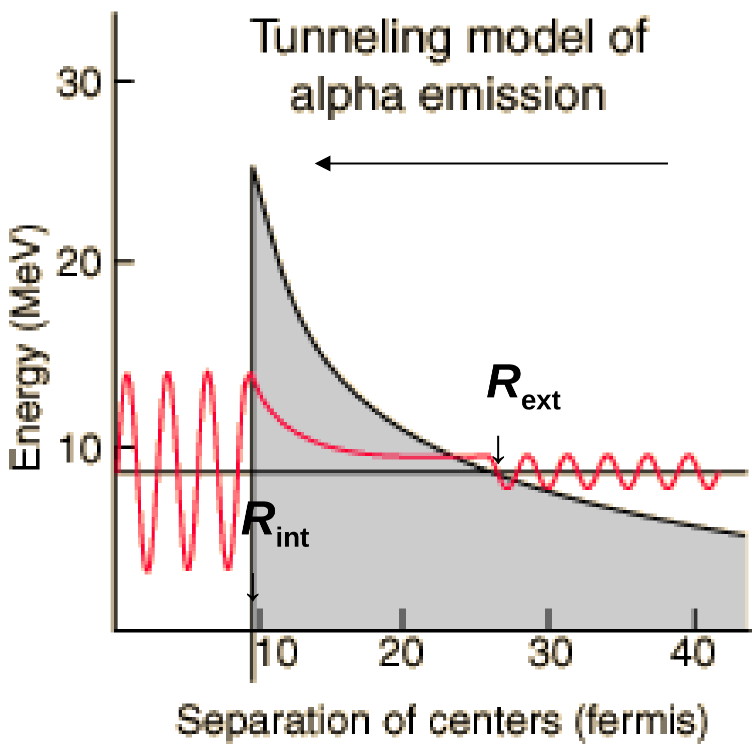
The first probabilistic interpretation of the wave function
for a particle penetrating a repulsive Coulomb barrier was given by

G. Gamow "Zur Quantentheorie des Atomkernes" (On the quantum theory of the atomic nucleus), *Zeitschrift für Physik*, vol. 51, 204-212 (1928).

The largest value of the Coulomb barrier:

$$B = V(R_{\text{int}}) = \frac{Z_D Z_C e^2}{R_{\text{int}}}$$

$$\Psi_{\text{int}} \rightarrow$$



$$\leftarrow N\Psi_{ext}$$

Internal region **External region**

Geiger-Nuttall law in terms of the Coulomb parameter

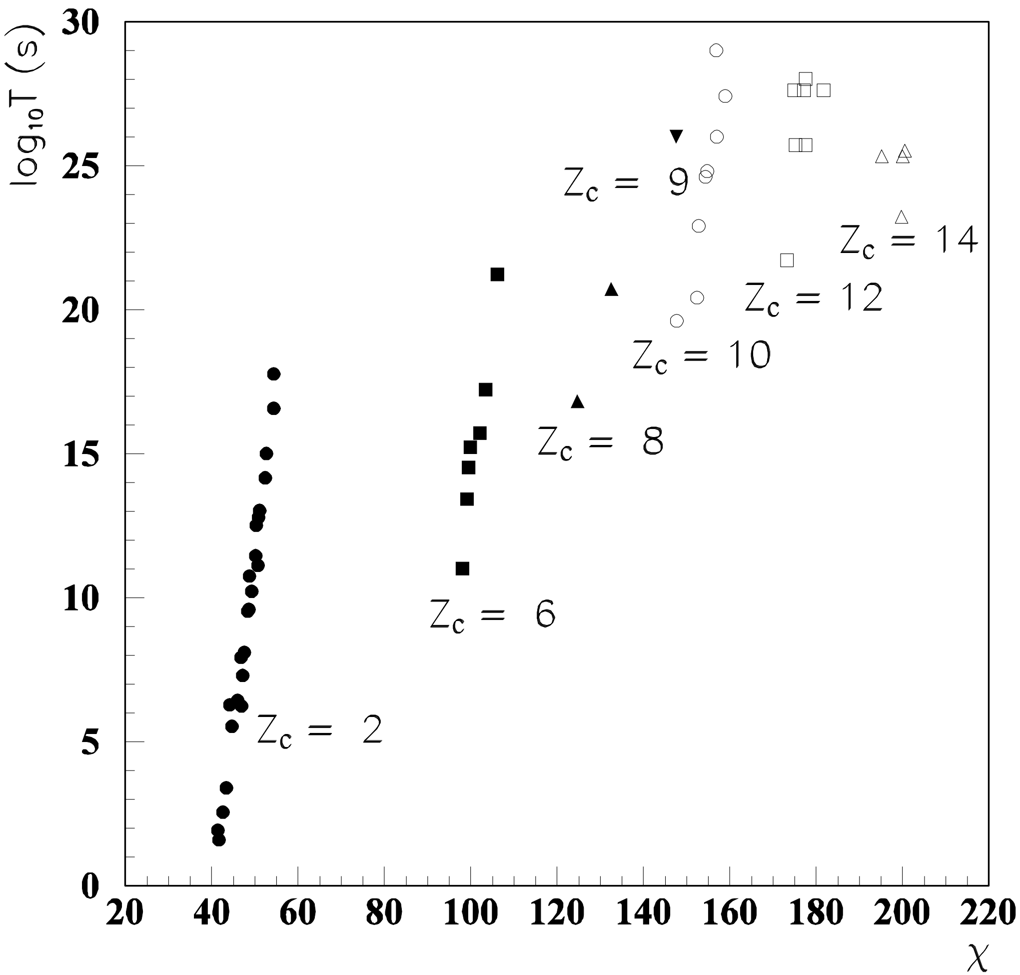
$$\log_{10}\Gamma = a\chi + b$$

where the Coulomb parameter is:

$$\chi = \frac{2Z_D Z_C}{\hbar v} \propto \frac{2Z_D Z_C}{\sqrt{E}}$$

$$Z_C = 2$$

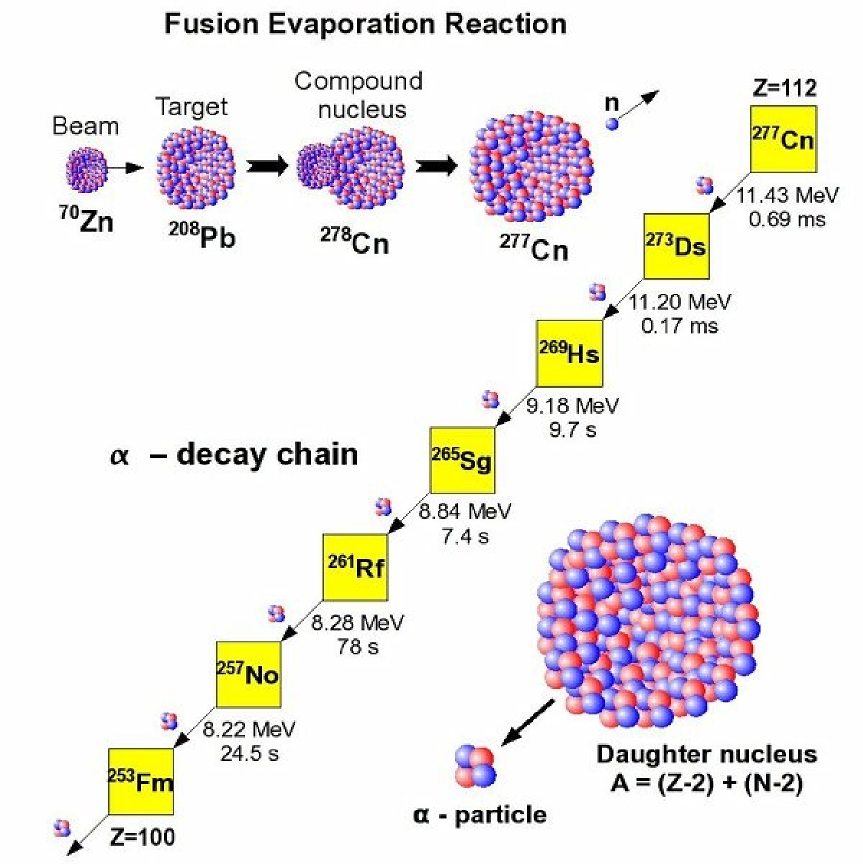
Quantum penetration explains Geiger-Nuttall law for α & cluster decays (emission of C, O, Ne, Mg, Si)



Coulomb parameter
In cluster emission:

$$\chi = \frac{2 Z_D Z_C e^2}{\hbar v} \rightarrow c \frac{Z_D Z_C}{\sqrt{E}}$$

Superheavy nuclei
are formed by fusion
and detected by alpha decay chains



D.04. Beta decay

$$\beta^- : \quad n \rightarrow p + e^- + \bar{\nu}_e$$

Decay of a free neutron. ‘Lifetime’ ~ 886 s

Electron-anti-neutrino

$$\beta^+ : \quad p \rightarrow n + e^+ + \nu_e$$

Electron-neutrino

Cannot occur for a FREE proton ….. ?

Neutrino

$$p \rightarrow n + e^+$$

½ ½ ½

- Three fermions: all have spin = ½
- Spin angular momentum is not conserved in the above reaction
- The ‘other’ particle must have spin = ½

Theory of β-decay

Describe the strength of the interaction with an operator ('potential') V

Define a "matrix element"

$$V_{fi} = \int \psi_f^* V \psi_i \, dv$$

The 'initial' and 'final' states are coupled by the operator V

Fermi's Golden Rule

The transition probability is given by

$$\lambda_{fi} = \frac{2\pi}{\hbar} \left| V_{fi} \right|^2 \rho_f$$

The nature of the interaction was unknown at the time !

The density of states can be calculated in 'Phase Space' : a 6-dim space defined by 3 spatial coordinates and 3 momentum coordinates

$$\rho_f = \frac{dn}{dE_f} = \frac{dn}{dp} \cdot \frac{dp}{dE} \quad \propto p_e^2$$

Fermi's Golden Rule

The beta-decay must include both the electron and the associated anti-neutrino

$$\frac{d^2 n}{dp_e dp_{\bar{\nu}}} = \propto p_e^2 p_{\bar{\nu}}^2$$

$$E = E_e + E_\nu = E_e + p_\nu c$$

$$dp_\nu = {dE}/{c}$$

$$\therefore \rho \propto p_e^2 (E - E_e)^2$$

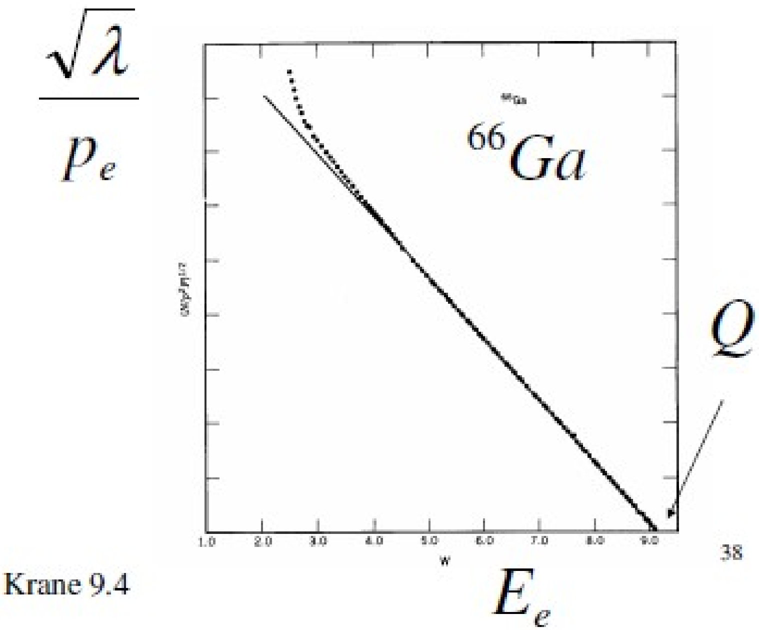
Ignores recoil KE of daughter atom.
Assumes neutrino has zero mass

(Fermi-) Kurie plot

The transition rate is therefore:

$$\lambda \propto p_e^2 (E - E_e)^2$$

$$\therefore \quad \frac{\sqrt{\lambda}}{p_e} \propto (E - E_e)$$



Krane 9.4

Fermi & Gamow-Teller transitions

Fermi decay: electron and anti-neutrino spins are antiparallel

$$\Delta I = 0 \quad \& \quad \Delta\pi = No$$

Gamow-Teller decay: electron and anti-neutrino spins are parallel

$$\Delta I = 0, \pm 1 \quad \& \quad \Delta\pi = No$$

Except $0 \rightarrow 0$

This assumes that the electron and the anti-neutrino do not carry orbital angular momentum

Calculate the decay rate using the Fermi or the Gamow-Teller mechanisms.
Mirror-decays are mixed transitions:

$$n \rightarrow p + e^{-} + \overline{\nu}_e$$

$$18\%\ F + 82\%\ GT$$

$${}^{13}_{7}N \rightarrow {}^{13}_{6}C \qquad 76\%\ F + 24\%\ GT$$

Non-Mirror-decays are GT-dominant:

$$\geq 98\%\ GT$$

β decay is induced by the weak interaction

β-decay mechanism ?

FERMI model assumed a pure VECTOR interaction

$$\Delta I = 0 \quad \& \quad \Delta\pi = No$$

GAMOW-TELLER assumed a TENSOR or AXIAL interaction. Takes into account the spins.

$$\Delta I = 0, \pm 1 \quad \& \quad \Delta\pi = No$$

Now we know that the Nuclear WEAK interaction is Vector-Axial. Strength is about 10^{-6} of the Nuclear Strong force.

An application of β-decay "Carbon Dating"

- The ratio of ^{14}C to ^{12}C is ~ 1.3 x 10^{-12}
- Living organisms exchange CO_2 with surroundings so ratio is ~ stable.
- Exchange stops at death so ratio ↓ because the ^{14}C decays.

$$^{14}_{6}C \rightarrow ^{14}_{7}N + e^{-} + \overline{\nu}_e$$

$$t_{1/2} = 5730\ yrs$$

25 g charcoal
^{14}C activity = 250 decays/min
Decay constant

$$\lambda = \frac{\ln 2}{t_{1/2}} = 3.83\times10^{-12}\ s^{-1}$$

Number of ^{12}C nuclei in 25 g

$$= 6.02\times10^{23}\times\frac{25}{12} = 1.26\times10^{24}$$

$$N_o\left(^{14}C\right) = 1.26\times10^{24}\times1.3\times10^{-12} = 1.6\times10^{12}$$

Initial activity (at death)

$$R_o = \lambda N_o = 6.13\, decays/\sec = 370\, decays/\min$$

$$R = \lambda N_0 e^{-\lambda t}$$

$$250 = 370 e^{-\lambda t} \rightarrow t = 3239\ yrs$$

D.05. Gamma decay

- Transition between nuclear states: $$I_i \xrightarrow{\gamma} I_f$$
- A multipole of order L transfers $L\hbar$ angular momentum per photon

$$\vec{I}_i = \vec{L} + \vec{I}_f$$

$$e.g. \quad (I_i, I_f) = \left(\tfrac{3}{2}, \tfrac{5}{2}\right) \rightarrow L = 1, 2, 3, 4$$

$$i.e. \quad \left|I_i - I_f\right| \leq L \leq \left(I_i + I_f\right)$$

- ‘Electric’ or ‘Magnetic’ depends on parities of nuclear states

Parity rules for gamma transitions

- Multipole Radiation: Electric and Magnetic
- Opposite parities

$$\pi(EL) = (-1)^L \quad \& \quad \pi(ML) = (-1)^{L+1}$$

- $L = 1 \longrightarrow$ Dipole
- $L = 2 \longrightarrow$ Quadrupole
- $L = 3 \longrightarrow$ Octupole
- $L = 4 \longrightarrow$ Hexadecapole etc

- Transition between nuclear states: $I_i \xrightarrow{\gamma} I_f$
- A multipole of order L transfers $L\hbar$ angular momentum per photon

$$\vec{I}_i = \vec{L} + \vec{I}_f$$

$$e.g. \quad (I_i, I_f) = \left(\tfrac{3}{2}, \tfrac{5}{2}\right) \rightarrow L = 1, 2, 3, 4$$

$$i.e. \quad \left|I_i - I_f\right| \le L \le \left(I_i + I_f\right)$$

- ‘Electric’ or ‘Magnetic’ depends on parities of nuclear states

Decay operators
in second quantisation:

gamma transitions

$$\hat{\boldsymbol{\gamma}} = \sum_{\tau=p,n} e_\tau \sum_{f,i} \langle \tau, f \,|\, \hat{V}_{\lambda\mu} \,|\, \tau, i \rangle \hat{\mathbf{a}}^+_{\tau f} \hat{\mathbf{a}}_{\tau i}$$

$$\hat{V}_{\lambda\mu} = r^\lambda Y_{\lambda\mu}$$

beta transitions

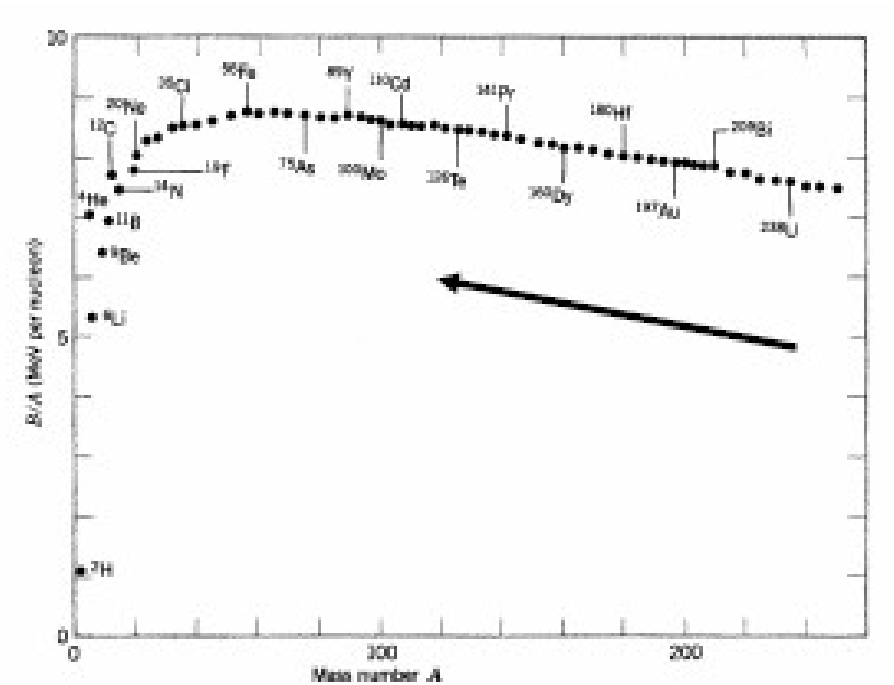
$$\hat{\boldsymbol{\beta}}^- = \sum_{f,i} \langle p, f \,|\, \hat{V}_{\lambda\mu} \,|\, n, i \rangle \hat{\mathbf{a}}^+_{pf} \hat{\mathbf{a}}_{ni}$$

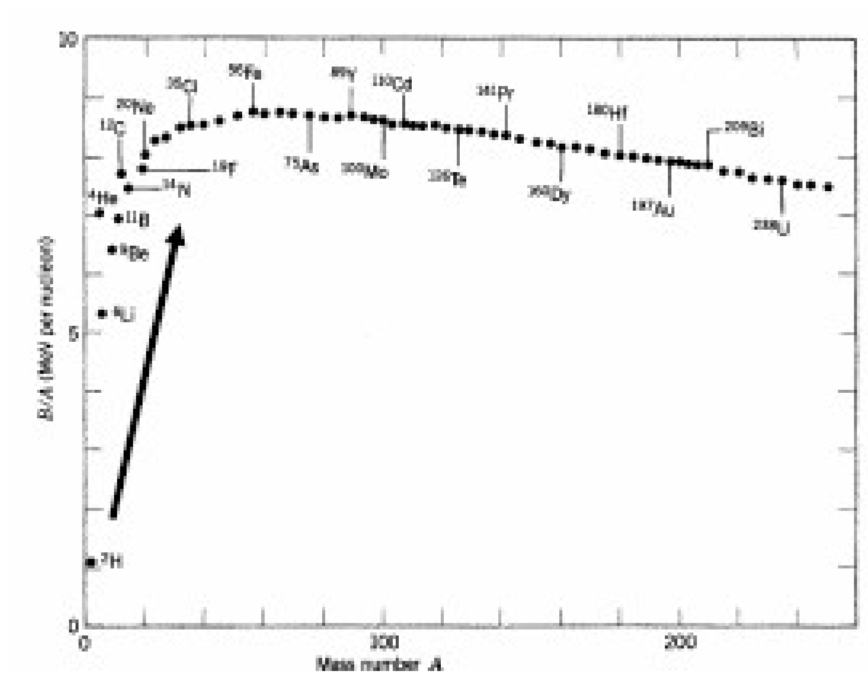
$$\hat{\boldsymbol{\beta}}^+ = \sum_{f,i} \langle n, f \,|\, \hat{V}_{\lambda\mu} \,|\, p, i \rangle \hat{\mathbf{a}}^+_{nf} \hat{\mathbf{a}}_{pi}$$

$$\hat{V}_F = 1$$

$$\hat{V}_{GT} = \sigma$$

D.06. Fission & fusion are mirror processes





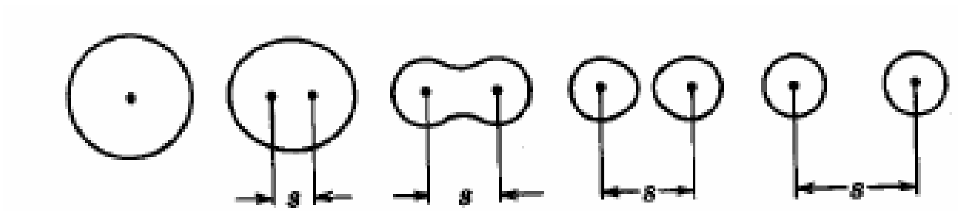
Fissioning nuclei produce lighter fragments + energy

Fusioning nuclei produce heavier fragments + energy

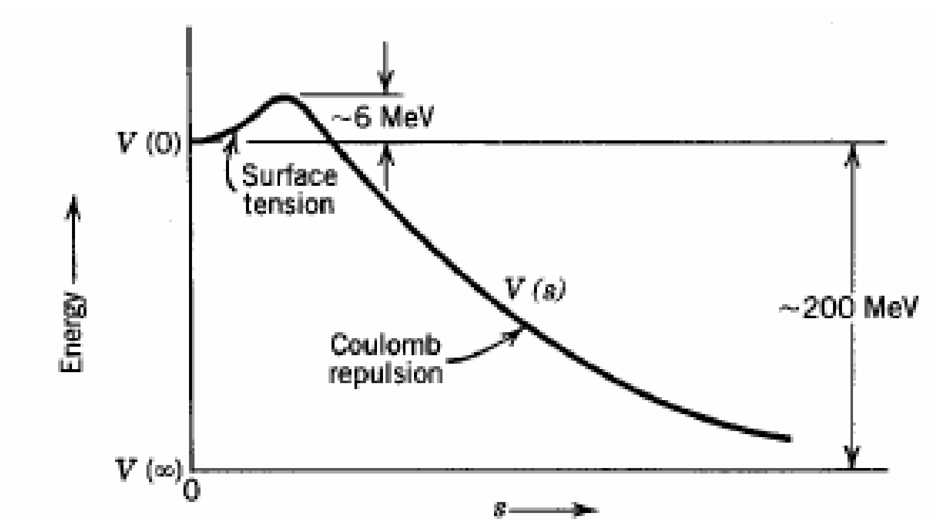
Spontaneous fission
liquid drop description

Bohr and Wheeler

Dynamics of the liquid drop during the fission process



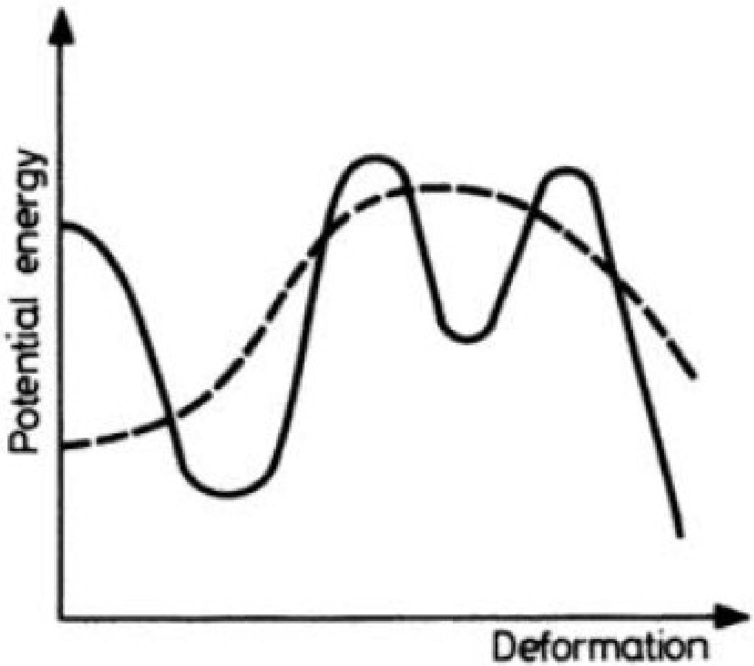
Fission barrier →



The decay width is given by the penetrability through the Coulomb barrier (as in the α decay theory)

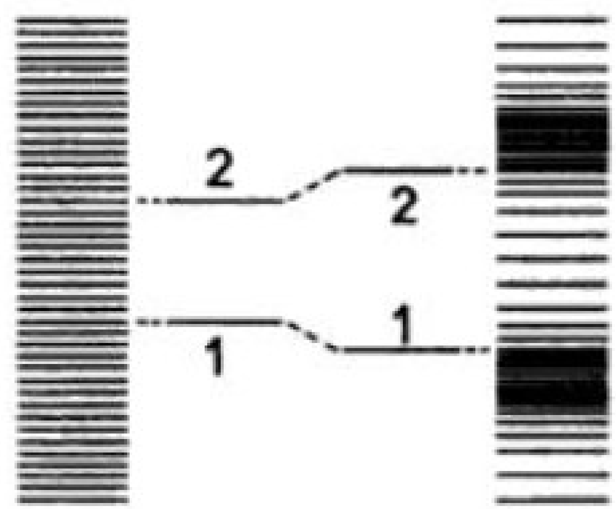
Strutinsky method
to compute the fission barrier:

the sum between the liquid drop energy and SM correction (the difference between SM energy and its mean value).



Density of levels

liquid drop shell model



Fusion energy

- To fuse two protons, one must overcome the Coulomb repulsion (barrier)
- e.g. two protons 5 fm apart

$$U = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= (9\times10^9)(1.6\times10^{-19})^2/(5\times10^{-15})$$

$$= 4.6\times10^{-14}\,J \quad (288\,keV)$$

- We have to provide thermal energy of 144 keV to each proton

$$\langle K \rangle = \tfrac{3}{2}k_B T = 2.4\times10^{-14}\,J$$

$$\therefore \quad T = \tfrac{2}{3}(2.4\times10^{-14}\,J)/(1.381\times10^{-23}\,J/K)$$

$$= \quad 1.2\times10^9\,K$$

Energy release for various processes

- Coal (1 kg) $\rightarrow$ 3×10^7 J
- Oil (1 barrel/0.16 m^3) $\rightarrow$ 6×10^9 J
- Natural Gas (1 m^3) $\rightarrow$ 4×10^7 J
- Uranium (1 kg) $\rightarrow$ 10^{14} J
- Deuterium fusion (1 kg) $\rightarrow$ 2×10^{14} J

The Sun

- The temperature of the solar core is about 15 million K.

- The nuclear fusion mechanism in The Sun is the proton-proton cycle.

$$p + p \rightarrow d + e^{+} + \nu_e \qquad Q = 0.42 \text{ MeV}$$

$$p + d \rightarrow {}^3_2He + \gamma \qquad Q = 5.49 \text{ MeV}$$

$${}^3_2He + {}^3_2He \rightarrow p + p + \alpha \qquad Q = 12.86 \text{ MeV}$$

•The net result is the ‘burning’ of 4 protons to form an alpha particle + positrons, neutrinos, gammas (Eddington).