Lecture I.1
Nuclear Structure Observables

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Bibliography

- G. Vladuca – Elemente de fizica nucleara
- A. Bohr, B. Mottelson – Nuclear structure
- K. Heyde – Basic idea and concepts in nuclear physics
- R. Casten – Nuclear structure from a simple perspective
- Wikipedia

Outline

- Physical Observables;
- Spectra and histograms; Nuclear spectra;
- Continuous and discrete spectra;
- Statistics; The *Gauss distribution*;
- The central limit theorem

Physical observables

Avoiding any formula from quantum mechanics, in the present course, by *physical observable* we mean any propriety of the nucleus that can be measured or determined.

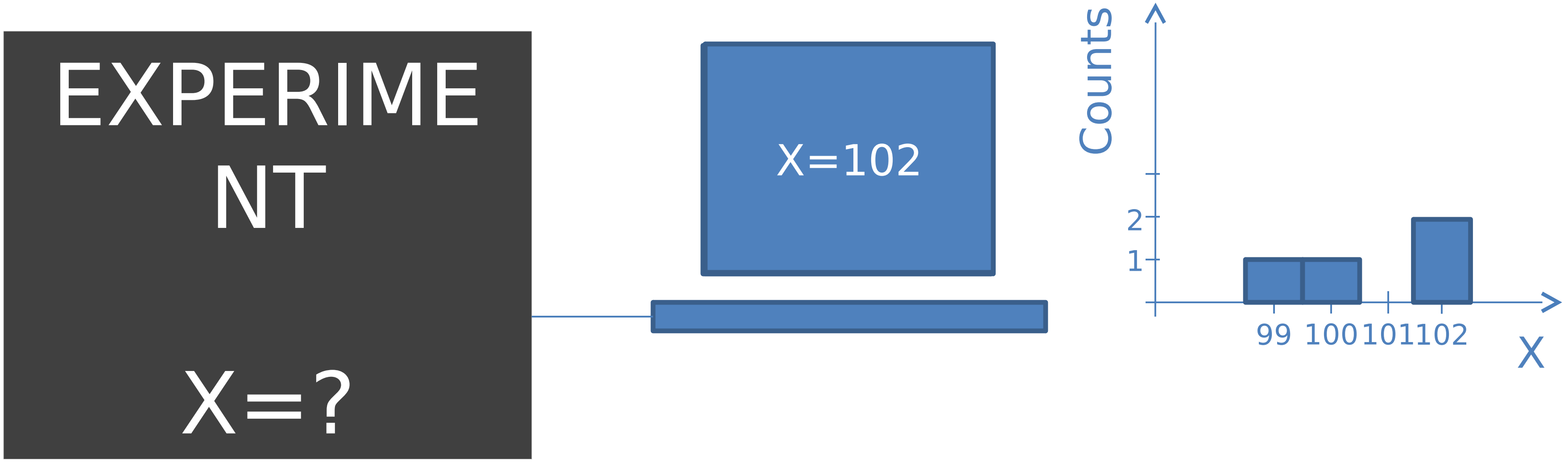
We include here things like the level energies, spin and parities, branching ratios, etc.

We exclude things like interaction potentials, wave functions, etc.

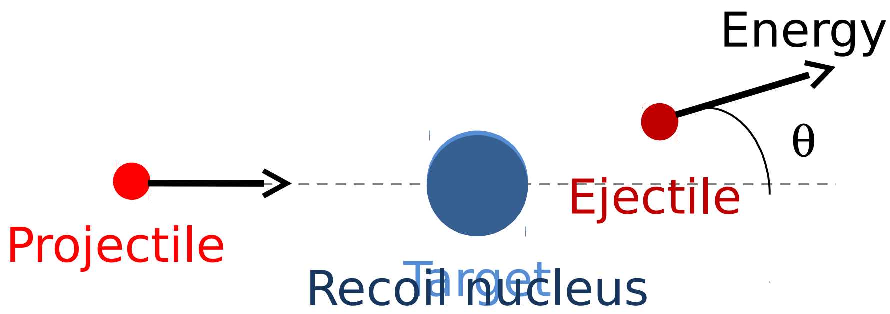
Spectra and histograms

The atomic nucleus is a quantum system. In quantum mechanics all phenomena are described by probabilities.

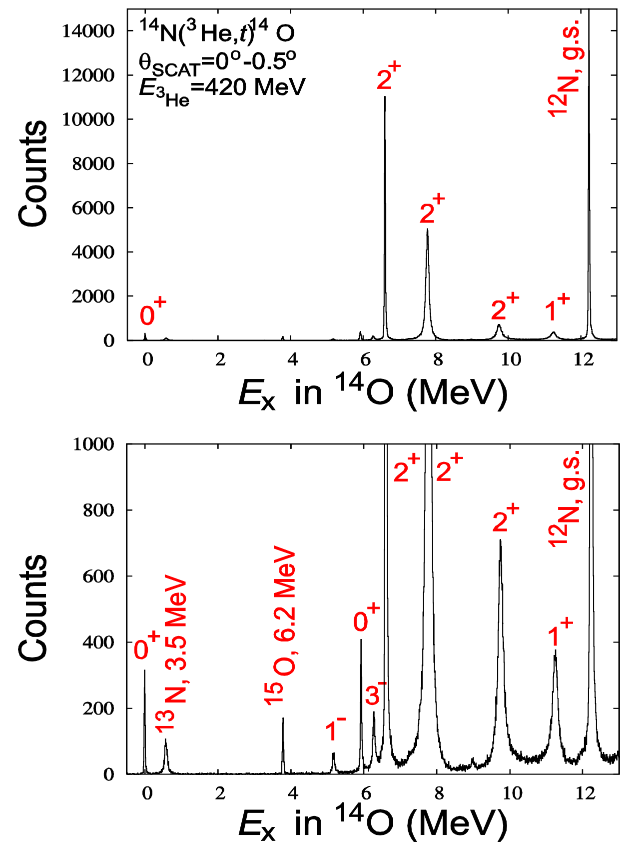
In order to record probabilities we need spectra. A spectrum is a representation of an observable that can be histogramed:



Examples of spectra
Continuous and discrete spectra



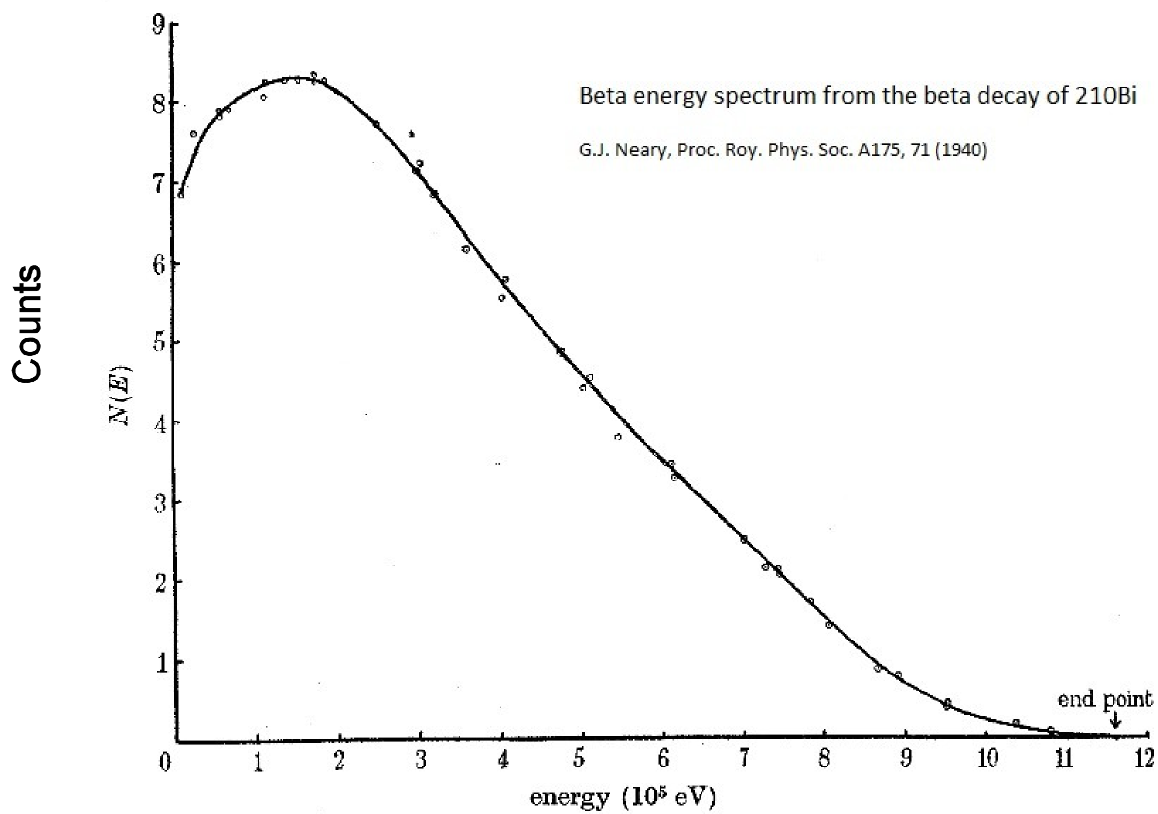
Energy spectrum



Exercise: continuous or discrete?

The gamma spectrum of a nucleus excited in a nuclear reaction

The beta spectrum of a nucleus undergoing beta decay



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A few more words on statistics

Nuclear processes are of statistical nature. Therefore we will have to deal with distributions, averages, uncertainties, etc.

We measure x and y *n* times:
$x_1, x_2, \ldots x_i, \ldots x_n$ and $y_1, y_2, \ldots y_i, \ldots y_n$.
Then:

Expected value:

$$E[x] = \frac{1}{n}\sum_{i=1}^{n} x_i$$

n is very large.

Covariance:

Cov(x,y) = E[(x-E(x))(y-E[y])]

Variance:

Var(x) = Cov(x,x) = σ^2

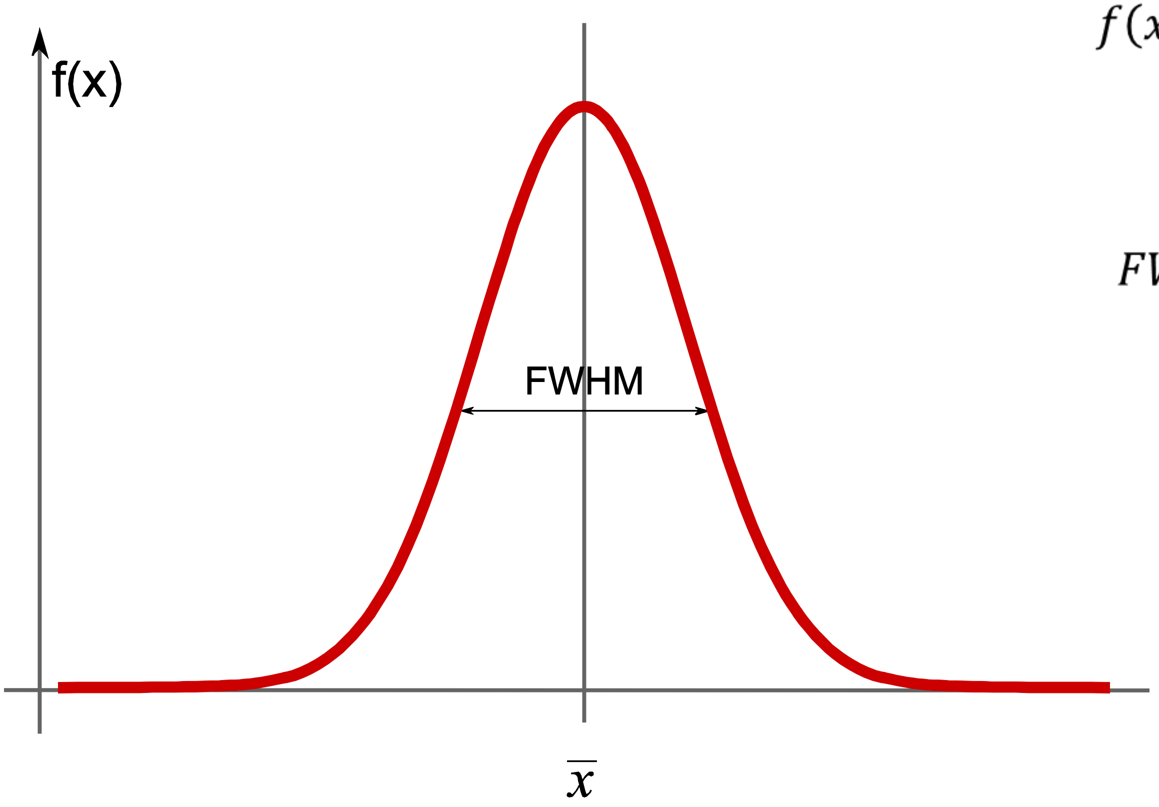
Standard deviation; (sometimes uncertainty)

Error propagation f(x,y):

$$\sigma^2(f) = \left(\frac{\partial f}{\partial x}\right)^2 \sigma^2(x) + \left(\frac{\partial f}{\partial y}\right)^2 \sigma^2(y) + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} Cov(x,y)$$

The Gauss distribution

The Gauss (Normal) distribution models the detection resolut
of many experimental devices.



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$

$$FWHM = 2\sqrt{2\ln 2}\,\sigma$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Question: Why did I discuss the Gauss Distribution?

he Gauss distribution models the detection resolution of many experimental dev
/hy?

Central limit theorem:

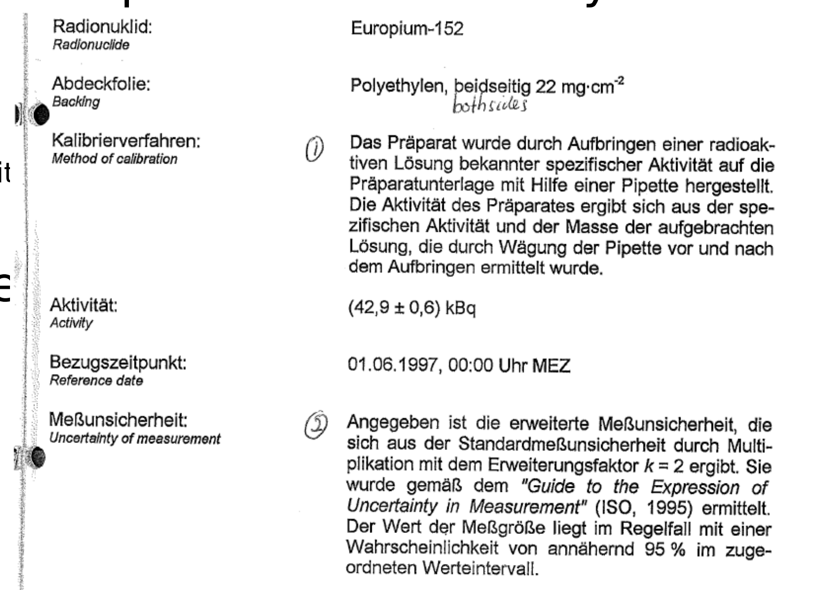
The sum of a large number of random variables has a normal distribution.

As an experimentalist, if you make sufficient randomly distributed mistakes, the average of your results will be the correct result!!!

Also: if the experimental setup is sufficiently complicated and each component introduces an error, the final result is distributed on a Gauss distribution and the average value is the real experimental value.

Exercise: Uncertainty for an efficiency determination

- We make an measurement and we count $N_{detected}$ gamma rays.
- The producer of the gamma source provides the activity of the source at a reference date
- Efficienciency: $\varepsilon = N_{detected} / N_{emit}$
- How do we practically calculate



Radionuklid: *Radionuclide*	Europium-152
Abdeckfolie: *Backing*	Polyethylen, beidseitig 22 mg·cm^{-2} (both sides)
Kalibrierverfahren: *Method of calibration*	(1) Das Präparat wurde durch Aufbringen einer radioaktiven Lösung bekannter spezifischer Aktivität auf die Präparatunterlage mit Hilfe einer Pipette hergestellt. Die Aktivität des Präparates ergibt sich aus der spezifischen Aktivität und der Masse der aufgebrachten Lösung, die durch Wägung der Pipette vor und nach dem Aufbringen ermittelt wurde.
Aktivität: *Activity*	(42,9 ± 0,6) kBq
Bezugszeitpunkt: *Reference date*	01.06.1997, 00:00 Uhr MEZ
Meßunsicherheit: *Uncertainty of measurement*	(2) Angegeben ist die erweiterte Meßunsicherheit, die sich aus der Standardmeßunsicherheit durch Multiplikation mit dem Erweiterungsfaktor $k = 2$ ergibt. Sie wurde gemäß dem *"Guide to the Expression of Uncertainty in Measurement"* (ISO, 1995) ermittelt. Der Wert der Meßgröße liegt im Regelfall mit einer Wahrscheinlichkeit von annähernd 95 % im zugeordneten Werteintervall.

Exercise: Uncertainty for an efficiency determination

$$\varepsilon = \frac{N_{detected}}{N_{emitted}} = \frac{N_d}{N_e}$$

$$\sigma_\varepsilon^2 = \left(\frac{\partial \varepsilon}{\partial N_d}\right)^2 \sigma_{N_d}^2 + \left(\frac{\partial \varepsilon}{\partial N_e}\right)^2 \sigma_{N_e}^2 = \left(\frac{1}{N_e}\right)^2 \sigma_{N_d}^2 + \left(\frac{-N_d}{N_e^2}\right)^2 \sigma_{N_e}^2$$

$$\sigma_{N_d} = \sqrt{N_d}$$

$N_e, \sigma_{N_e} = ?$

The source producer provides the activity Λ_0 at a reference date t_0 and the decay constant of the source

$$\Lambda = \Lambda_0 e^{-\lambda t} \qquad N_e = T\Lambda = T\Lambda_0 e^{-\lambda t}$$

- T: Measurement time
- t: Time difference between t_0 and the moment of measurement

$$\sigma_{N_e}^2 = \left(\frac{\partial N_e}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial N_e}{\partial \Lambda_0}\right)^2 \sigma_{\Lambda_0}^2 + \left(\frac{\partial N_e}{\partial \lambda}\right)^2 \sigma_\lambda^2 + \left(\frac{\partial N_e}{\partial t}\right)^2 \sigma_t^2$$

$$= \left(\Lambda_0 e^{-\lambda t}\right)^2 \sigma_T^2 + \left(T e^{-\lambda t}\right)^2 \sigma_{\Lambda_0}^2 + \left(tT\Lambda_0 e^{-\lambda t}\right)^2 \sigma_\lambda^2 + \left(\lambda T \Lambda_0 e^{-\lambda t}\right)^2 \sigma_t^2$$

- σ_T: 1s ?
- σ_{Λ_0}: Given by the producer
- σ_λ: From a database (possibly negligible)
- σ_t: Comparable with T?

Summary

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