SCIENTIFIC REPORT

Implementation of the project IDEI 42/05.10.2011
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Binary nuclear systems

Objective: *Pairing corrections and inertia tensors in binary nuclear systems*

The main objective of the project is the selection of the optimal target-projectile pairs in sub-barrier fusion. These reactions will be used in the synthesis of superheavy nuclei. The sub-barrier fusion reaction, though it occurs with a lower cross section, has the advantage of a final state closer to the ground state. Consequently one hopes for a longer lifetime of the nucleus.

Superheavy nuclei are stable only due to the shell and pairing corrections. The macroscopic energy produces no barrier. We calculated here the pairing corrections within a specialized binary model, the deformed two center shell model. The pairing corrections have been obtained by solving the BCS system, in order to obtain the Fermi level for paired nucleons and the energy gap which appears when protons and neutrons are under pairing interaction. The total microscopic correction is the sum of the shell E_{shell} and pairing δP energies:

$$\delta E = E_{shell} + \delta P \tag{1}$$

The pairing part is calculated by the difference between the sum of the simple particle levels with and without pairing introduced, and the subtraction of a uniform distribution pairing $\tilde{P}$:

$$\delta P = P - \tilde{P} \tag{2}$$

The energy due to the nucleon pairing, P is:

$$p = \sum_{k=k_i}^{k_f} 2v_k^2\epsilon_k - 2\sum_{k=k_i}^{Z/2} \epsilon_k - \frac{\Delta^2}{G} \tag{3}$$

The uniform distribution part $\tilde{P}$ reads:

$$\tilde{p} = -(\tilde{g}\tilde{\Delta}^2)/2 = -(\tilde{g}_s\tilde{\Delta}^2)/4 \tag{4}$$

Compared to the shell correction, the pairing energy is in antiphase and smaller. When we have paired nucleons, they have an occupation v_k^2 and non-occupation u_k^2 probability different from 1. These probabilities depend on the energy gap Δ and the new Fermi level λ:

$$v_k^2 = \left[1 - (\epsilon_k - \lambda)/E_k\right]/2 \tag{5}$$
$$u_k^2 = 1 - v_k^2 \tag{6}$$

These quantities determine the inertia tensor, which is necessary in the dynamics of the process via the action integral. In order to obtain the mass tensor components, one starts from the kinetic energy of the collective motion:

$$T = \frac{1}{2}\sum_{i,j} B_{ij}(\vec{q})\dot{q}_i\dot{q}_j \tag{7}$$

where the mass parameters B_{ij} are structure dependent:

$$B_{ij} = 2\hbar^2 \sum_m \frac{\langle 0|\partial/\partial q_i|m\rangle\langle m|\partial/\partial q_j|0\rangle}{E_m + E_0} \tag{8}$$

In the quasiparticle interpretation we have:

$$B_{ij} = 2\hbar^2 \sum_{kk'} \frac{\langle k'|\partial H_{DTCSM}/\partial q_i|k\rangle\langle k|\partial H_{DTCSM}/\partial q_j|k'\rangle}{(E_k + E_{k'})^3}(u_k v_{k'} + u_{k'} v_k)^2 + P_{ij} \tag{9}$$

The binary character specific to this project appears in the use of the deformed two center Hamiltonian H_{DTCSM}. The deformation parameters are: the ratios of the semiaxes $\chi_1 = b/a_1$ and $\chi_2 = b_2/a_2$, the small semiaxis of the projectile b_2 and the distance between centers R. Finally we obtained the components of the tensor:

$$B_{\chi_1\chi_1} = 2\hbar^2 \sum_{kk'} \frac{\langle k'|\partial V_1/\partial \chi_1|k\rangle\langle k|\partial V_1/\partial \chi_1|k'\rangle}{(E_k + E_{k'})^3}(u_k v_{k'} + u_{k'} v_k)^2 + P_{kk'} \tag{10}$$

(11)

$$B_{\chi_1\chi_2} = 2\hbar^2 \sum_{kk'} \frac{\langle k'|\partial V_1/\partial \chi_1|k\rangle\langle k|\partial V_2/\partial \chi_2|k'\rangle}{(E_k + E_{k'})^3}(u_k v_{k'} + u_{k'} v_k)^2 + P_{kk'} \tag{12}$$

(13)

$$B_{\chi_1 R} = 2\hbar^2 \sum_{kk'} \frac{\langle k'|\partial V_1/\partial \chi_1|k\rangle\langle k|\partial V_2/\partial R|k'\rangle}{(E_k + E_{k'})^3}(u_k v_{k'} + u_{k'} v_k)^2 + P_{kk'} \tag{14}$$

(15)

$$B_{\chi_2\chi_2} = 2\hbar^2 \sum_{kk'} \frac{\langle k'|\partial V_2/\partial \chi_2|k\rangle\langle k|\partial V_2/\partial \chi_2|k'\rangle}{(E_k + E_{k'})^3}(u_k v_{k'} + u_{k'} v_k)^2 + P_{kk'} \tag{16}$$

(17)

$$B_{\chi_2 R} = 2\hbar^2 \sum_{kk'} \frac{\langle k'|\partial V_2/\partial \chi_2|k\rangle\langle k|\partial V_2/\partial R|k'\rangle}{(E_k + E_{k'})^3}(u_k v_{k'} + u_{k'} v_k)^2 + P_{kk'} \tag{18}$$

(19)

$$B_{RR} = 2\hbar^2 \sum_{kk'} \frac{\langle k'|\partial V_2/\partial R|k\rangle\langle k|\partial V_2/\partial R|k'\rangle}{(E_k + E_{k'})^3}(u_k v_{k'} + u_{k'} v_k)^2 + P_{kk'} \tag{20}$$

In order to use the tensor in the calculation, one submit the components to a contraction along the R variable. In this way one obtains:

$$B(R) = B_{b_2,b_2}\left(\frac{b_2}{dR}\right)^2 + 2B_{b_2\chi_1}\frac{db_2}{dR}\frac{d\chi_1}{dR} + 2B_{b_2\chi_2}\frac{b_2}{dR}\frac{d\chi_2}{dR} + 2B_{b_2R}\frac{db_2}{dR} \tag{21}$$

$$+ \; B_{\chi_1\chi_1}\left(\frac{\chi_1}{dR}\right)^2 + 2B_{\chi_1\chi_2}\frac{d\chi_1}{dR}\frac{d\chi_2}{dR} + 2B_{\chi_1 R}\frac{d\chi_1}{dR} \tag{22}$$

$$B_{\chi_2\chi_2}\left(\frac{\chi_2}{dR}\right)^2 + 2B_{\chi_2 R}\frac{d\chi_2}{dR} + B_{RR} \tag{23}$$

These values enter in the action integral formula:

$$K_{ov}(b_P, \kappa_T, \kappa_P; R) = \frac{2}{\hbar}\int_{(fus)} [2B(R)_{b_P,\kappa_T,\kappa_P} E_{def}(R)_{b_P,\kappa_T,\kappa_P}]^{1/2} dR \tag{24}$$

The total penetrability in the sub-barrier fusion process is:

$$P = \exp(-K_{ov}) \tag{25}$$

Calculations have been performed for superheavy nuclei. We obtained for example the most favorable reaction ^{160}Yb+^{132}Sn $\rightarrow$ ^{292}120 (highest penetrability).

The results have been published in ISI journals:
ISI articles:

1. D. N. Poenaru, R. A. Gherghescu, W. Greiner
   J. Phys. G40, 105105 (2013).
2. R. A. Gherghescu, D. N. Poenaru
   Rom. J. Phys. 58, 1178 (2013).
3. R. A. Gherghescu,D. N. Poenaru, W. Greiner
   invited lecture in Exciting Interdisciplinary Physics, FIAS (Springer, Heidelberg 2013), p. 129.

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