Raport stiintific privind implementarea proiectului in perioada septembrie 2013-decembrie 2014

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I. RAPORT STIINTIFIC SEPTEMBRIE 2013-DECEMBRIE 2014

In cadrul proiectului au fost trimise in total spre publicare noua lucrari dintre care trei au fost acceptate spre publicare in Int. J. Mod. Phys. A, doua au fost publicata in Mod. Phys. Lett. A, si patru sunt in evaluare.

In cadrul etapei unice a proiectului pe anul 2013 au fost trecut umratorul obiectiv:

1) Indicatii despre confinement in cadrul unei noi simetrii a teoriilor de etalonare. In cadrul acestui obiectiv au fost propuse urmatoarele activitati.

a) Implementarea unei noi simetrii in teoriile de etalonare.

b) Aplicatii ale noii simetrii.

In cadrul etapei unice pe anul 2014 au fost propuse trei obiective:

1) Functia de partitie pentru o teorie scalara fara rupere spontana a simetriei intr-o noua metoda de integrare functionala. In cadrul acestui obiectiv au fost propuse doua activitati:

a) Stabilirea unei noi metode de integrare functionala.

b) Determinarea propagatorului scalar.

2) Functia de partitite in teoriile de etalonare cu fermioni intr-un nou formalism functional. In cadrul acestui obiectiv au fost propuse doua activitati:

a) Stabilirea unui nou cadru functional cu privire la functia de partitie.

b) Formule de corelare a functiilor de partitie in noul formalism.

3) Functia de partitite in teoriile de etalonare cu interactie tare intr-un nou formalism functional. In cadrul acestui obiectiv a fost propusa o singura activitate.

a) Aplicatii la teoriile de etalonare cu interactie tare.

Vom discuta rezultatele stiintifice obtinute pe obiective.

A. Indicatii despre confinement in cadrul unei noi simetrii a teoriilor de etalonare.

Pornind de la studiul operatorilor non-chirali care actioneaza intr-o teorie generala de etalonare am propus o noua simetrie care actioneaza asupra unui Lagrangian ce contine atat campuri de etalonare cat si fermioni si campuri scalare. Mai intai am introdus operatorul non-hermitian K,

$$K = \exp[k\gamma^{\mu} D_{\mu}] \tag{1.1}$$

ce actioneaza asupra fermionilor. Am demonstrat apoi ca un lagrangian invariant la actiunea unei simetrii de etalonare abeliene este de asemenea invariant la actiunea urmatoarelor transformari infinitezimale:

$$\begin{aligned}
\Psi' &= \Psi + k\gamma^\mu D'_\mu \Psi \\
A'_\mu &= A_\mu - \frac{1}{g}\alpha \\
B' &= B
\end{aligned} \tag{1.2}$$

cu conditia ca campul scalar B sa fie identificat pana la inversul unui 'scale factor" cu parametrul de etalonare:

$$\alpha = kB. \tag{1.3}$$

In esenta demonstratia este continuta in:

$$\begin{aligned}
\mathcal{L}' &= i\Psi^\dagger\gamma^0\gamma^\mu D'_\mu\Psi - \Psi^\dagger\gamma^0 B\Psi + ik(\gamma^\rho D'_\rho\Psi)^\dagger\gamma^0\gamma^\mu D'_\mu\Psi + \\
&+ik\Psi^\dagger\gamma^0\gamma^\mu D'_\mu\gamma^\rho D'_\rho\Psi - k(\gamma^\rho D_\rho\Psi)^\dagger\gamma^0 B\Psi - k\Psi^\dagger\gamma^0\gamma^\rho D_\rho\Psi = \\
&= i\Psi^\dagger\gamma^0\gamma^\mu D'_\mu\Psi + k\Psi^\dagger\gamma^0\gamma^\mu\partial_\mu B\Psi - \Psi^\dagger\gamma^0 B\Psi = \\
&= \mathcal{L} + gk\Psi^\dagger\gamma^0\gamma^\mu(A'_\mu - A_\mu)\Psi + k\Psi^\dagger\gamma^0\gamma^\mu\partial_\mu B\Psi.
\end{aligned} \tag{1.4}$$

Am demonstrat apoi ca acelasi tip de simetrie actioneaza si asupra unui lagrangian ce contine campuri de etalonare non-abeliene:

$$\begin{aligned}
&-(\gamma^\rho D_\rho\Psi)^\dagger\gamma^0 B\Psi - \Psi^\dagger\gamma^0\gamma^\rho D_\rho(B\Psi) = \\
&= \Psi^\dagger\gamma^0\gamma^\rho(D_\rho B)\Psi
\end{aligned} \tag{1.5}$$

De data aceasta obtinem ca,

$$g(A'_\mu - A_\mu) = -kD_\mu B. \tag{1.6}$$

Am aplicat aceleasi transformari tuturor termenilor din modelul standard al particulelor elementare dupa ruperea spontana a simetriei electroslabe si am demonstrat invarianta lagrangianului corespunzator. Aceasta noua simetrie are implicatii importante atat pentru sectorul corespunzator cromodinamicii cuantice cat si pentru cel al bozonului Higgs.

Simetria introdusa in Eq. (1.2) este o simetrie valida in forma ei infinitezimala deci pentru impulsuri $p \ll \frac{1}{k}$. Este bine stiut ca transformarile de etalonare sunt simetrii bune ale unui lagrangian abelian sau nonabelian atat in forma lor infinitezimala cat si cea finita. Simetria indusa de operatorul K' (care contine campurile de etalonare transformate) este foarte complicata in forma ei finita asa ca am studiat actiunea ei numai " on-shell" asupra unui lagrangian non-abelian, in speta QCD.

Cerem ca,

$$(K'\Psi^\dagger)\gamma^0(i\gamma^\mu D'_\mu - m - B)(K'\Psi) = \Psi^\dagger\gamma^0(i\gamma^\mu D_\mu - m - B)\Psi. \tag{1.7}$$

O posibila solutie e data de,

$$\begin{aligned}
&K'\Psi = \Psi' \\
&\ln(K')\Psi = i\alpha\Psi \\
&(-igk\gamma^\mu(A'_\mu - A_\mu) + ikm + ikB) = i\alpha\Psi.
\end{aligned} \tag{1.8}$$

Invarianta Lagrangianului la actiunea transformarilor induse de K' impune anumite constrangeri asupra campului scalar ce este proportional cu parametrul de etalonare si anume:

$$\partial^\mu B^a \partial_\mu B^a + g^2 f^{abc} A^{mub} B^c f^{amn} A^m_\mu B^n = \frac{m^2}{k^2}. \tag{1.9}$$

Am demonstrat ca campul scalar rezultat din Eq. (1.9) induce un potential de tip "confining" intre doi cuarci:

$$V(q) = -i\frac{y^2k^2}{m^2}\delta(\vec{q})' = \int(-i\frac{y^2k^2}{m^2})\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\vec{x}}\delta(\vec{q})' = -\frac{y^2k^2}{m^2}r \tag{1.10}$$

Realizarea simetriei K' in cadrul modelului standard al particulelor elementare duce la identificarea bozonului Higgs gasit la LHC cu $\frac{\alpha}{k}$ unde α este parametrul de etalonare al grupului de etalonare al interactiei electromagnetice. Pentru a fixa etalonarea trebuie sa facem o modificare a procedurii usuale si anume sa introducem o noua functionala generatoare:

$$\begin{aligned}
&Z[\bar{\Psi}, \Psi, A, B] = \text{const}\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}A\mathcal{D}B \exp[i\int d^4x\mathcal{L}] \times \\
&\exp[-i\int d^4x\frac{\omega^2}{2}]\delta(\partial^\mu\partial_\mu B + m^2B)\delta(\partial^\mu A_\mu - \omega)
\end{aligned} \tag{1.11}$$

Aceaata functionala situeaza bozonul Higgs "on-shell" cu o masa data de relatia:

$$m^2 = m_0^2(1 - \frac{k^2m_0^2}{g^2}) \tag{1.12}$$

Faptul ca bozonul Higgs nu poate participa decat ca stare initiala si finala intr-un proces este un rezultat important care va putea fi verificat sau respins de viitoare explorari experimentale.

B. Functia de partitie pentru o teorie scalara fara rupere spontana a simetriei intr-o noua metoda de integrare functionala

In prezent exista un anumit nivel de cunoastere care nu este complet cu privire la comportarea perturbativa a diverselor teorii, cum ar fi teoria Φ^4 sau teoriile de etalonare. De exemplu functiile beta pentru teoria Φ^4 sau pentru QED sunt cunoscute pana la ordinul cinci in timp ce pentru QCD sunt cunoscute pana la ordinul patru. Vom considera in cele ce urmeaza teoria Φ^4 ca un laborator pentru a studia o metoda de calcul functional care ulterior sa poata fi aplicata la modele mai sofisticate. Incepem cu Lagrangianul:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_1 \\
\mathcal{L}_0 &= \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} m_0^2 \Phi^2 \\
\mathcal{L}_1 &= -\frac{\lambda}{4!}\Phi^4.
\end{aligned} \tag{1.13}$$

Functionala generatoare dependenta de sursa externa J are expresia:

$$W[J] = \int d\Phi \exp[-\int d^4x[\frac{1}{2}(\frac{\partial \Phi}{\partial \tau})^2 + \frac{1}{2}(\Delta\Phi)^2 + \frac{1}{2}m_0^2\Phi^2 + \frac{\lambda}{4!}\Phi^4 + J\Phi]] \tag{1.14}$$

si poate fi scrisa ca:

$$W[J] = \exp[\int d^4x \mathcal{L}_1(\frac{\delta}{\delta J})] W_0[J] \tag{1.15}$$

unde,

$$W_0[J] = \int d\Phi \exp[\int d^4x(\mathcal{L}_0 + J\Phi)]. \tag{1.16}$$

Pentru a merge mai departe avem nevoie de urmatoarea identitate matematica:

$$\begin{aligned}
I \;&= \int dxdy\delta(x-y)exp[-af(x,y)] = \int dxdydz \exp[-i(x-y)z]\exp[-af(x,y)] = \\
&\int dxdydz \exp[-i(x-y)z - af(x,y)]
\end{aligned} \tag{1.17}$$

Pentru $f(x,y)=x^2y^2$ se poate forma patratul perfect:

$$-ixz-ax^2y^2=-(\sqrt{a}xy+\frac{iz}{2\sqrt{a}y})^2-\frac{z^2}{4ay^2}. \tag{1.18}$$

Daca introducem aceasta expresie in Eq. (1.78) obtinem:

$$I=\text{const}\int d\frac{1}{\sqrt{a}y}dz\exp[-\frac{z^2}{4ay^2}]\exp[iyz] \tag{1.19}$$

In acest fel dezvoltarea in serie in $\frac{1}{a}$ are sens si se poate scrie:

$$I=\text{const}\int dxdz\frac{1}{\sqrt{a}y}[1-\frac{z^2}{4ay^2}+...]\exp[iyz] \tag{1.20}$$

Consideram functia de partitie pentru teoria Φ^4 fara surse:

$$W[0]=\int d\Phi\exp[i\int d^4x[\mathcal{L}_0+\mathcal{L}_1]] \tag{1.21}$$

Scriem eq. (1.21) in spatiul Minkowski ca:

$$\begin{aligned}
&W[0]=\int d\Phi d\Psi\delta(\Phi-\Psi)\exp[i\int d^4x[\mathcal{L}_0-\frac{\lambda}{8}\Phi^2\Psi^2]=\\
&\text{const}\int d\Phi d\Psi dK\exp[i\int d^4xK(\Phi-\Psi)]\exp[i\int d^4x[\mathcal{L}_0-\frac{\lambda}{8}\Phi^2\Psi^2]=\\
&\text{const}\int\frac{1}{\sqrt{\lambda}}d\Phi dK\exp[i\int d^4x\frac{2}{\lambda}K^2]\exp[i\int d^4xK\Phi^2]\exp[i\int d^4x\mathcal{L}_0]
\end{aligned} \tag{1.22}$$

Dupa anumite simplificari si calcule efectuate in spatiul Fourier pentru integrand se obtine:

$$\begin{aligned}
&\text{const}\int\frac{1}{\sqrt{\lambda}}d\Phi dK\exp[i\int d^4x\frac{2}{\lambda}K^2]\exp[i\int d^4xK\Phi^2]\exp[i\int d^4x\mathcal{L}_0]=\\
&=\int dK\exp[i\int d^4x\frac{2}{\lambda}K^2]\frac{1}{\det[\frac{K}{V^2}+\frac{1}{2V}[\frac{2K_0}{V}-(m_0^2-p_n^2)\delta_{2n+1,2n+1}+\delta_{2n+2,2n+2}]]^{1/2}}
\end{aligned} \tag{1.23}$$

In continuare se utilizeaza definitia standard a propagatorului campului scalar si rezultatele anterioare pentru a se obtine:

$$\begin{aligned}
&\langle\Omega|T\Phi(x_1)\Phi(x_2)|\Omega\rangle=\frac{\int\frac{1}{\sqrt{\lambda}}d\Phi dK\Phi(x_1)\Phi(x_2)\exp[i\int d^4x\frac{2}{\lambda}K^2]\exp[i\int d^4xK\Phi^2]\exp[i\int d^4x\mathcal{L}_0]}{\int\frac{1}{\sqrt{\lambda}}d\Phi dK\exp[i\int d^4x\frac{2}{\lambda}K^2]\exp[i\int d^4xK\Phi^2]\exp[i\int d^4x\mathcal{L}_0]}=\\
&\frac{1}{V^2}\sum_m\exp[-ip_m(x_1-x_2)]iV\frac{\frac{\delta}{\delta(m^2-p_m^2)}\int d\Phi dK\exp[i\int d^4x\frac{2}{\lambda}K^2]\exp[i\int d^4xK\Phi^2]\exp[id^4x\mathcal{L}_0]}{\int d\Phi dK\exp[i\int d^4x\frac{2}{\lambda}K^2]\exp[i\int d^4xK\Phi^2]\exp[id^4x\mathcal{L}_0]}
\end{aligned} \tag{1.24}$$

Intrucat cantitatea $m^2 - k_n^2$ apare numai in determinantul Eq. (1.23) calculam:

$$\frac{\delta}{\delta(m_0^2 - p_m^2)}[\det[\frac{K}{V^2} + \frac{1}{2V}[\frac{2K_0}{V} - (m_0^2 - p_n^2)\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2}]]]^{-1/2} =$$
$$-\frac{1}{2}[\det[\frac{K}{V^2} - \frac{1}{2V}[\frac{2K_0}{V} - (m_0^2 - p_n^2)\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2}]]]^{-1/2} \times$$
$$Tr[\frac{1}{\frac{K}{V^2} + \frac{1}{2V}[2\frac{K_0}{V} - (m_0^2 - p_n^2)(\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2})]}(-1)(\frac{1}{2V}(\delta_{2m+1,2m+1} + \delta_{2m+2,2m+2})] =$$
$$\text{const}\frac{1}{2}[\det[\frac{K}{V^2} - \frac{1}{2V}[2\frac{K_0}{V} - (m_0^2 - p_n^2)\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2}]]]^{-1/2}\frac{2}{\frac{2}{V}K_0 - (m_0^2 - p_m^2)} =$$
$$\text{const}[\det[\frac{K}{V^2} - \frac{1}{2V}[2K_0 - (m_0^2 - p_n^2)\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2}]]]^{-1/2}\frac{1}{\frac{2K_0}{V} - (m_0^2 - p_m^2)}. \tag{1.25}$$

In Eq. (1.25) primele trei randuri sunt rezultatul diferentierii unui determinant. Primul factor in randul trei al Eq. (1.25) contine modurile Fourier ale campului K cu impulsuri diferite de zero ($p_\mu \neq 0$) pe care le numim simplu K si pe cele cu impulsuri zero $p_\mu = 0$ pe care le numim K_0. In abordarea noastra modurile cu $p_\mu \neq 0$ sunt neimportante pentru urmatorul motiv. Consideram $K(x)$ ca o functie patrat integrabila in spatiul Hilbert care satisface:

$$\int d^4x K^2(x) = \frac{1}{V}\sum_p K(p)^2 < M, \tag{1.26}$$

unde M este o cantitate mare dar finita. Aceasta inseamna ca $\frac{K(p)}{V} < \frac{\sqrt{M}}{\sqrt{V}}$ deci este o cantitate neglijabila pentru V foarte mare. In contrast $\frac{K_0}{V}$ este finita si este data de:

$$\frac{K_0}{V} = \int d^4x K(x). \tag{1.27}$$

In continuare neglijam toate modurile $p_\mu \neq 0$ si integram numai dupa modul K_0. Un scurt calcul conduce la:

$$\langle\Omega|T\Phi(x_1)\Phi(x_2)|\Omega\rangle = \frac{1}{V^2}\sum_m \exp[-ip_m(x_1 - x_2)]iV \times$$
$$\times\frac{\int dK \frac{1}{\frac{2}{V}K_0 - (m_0^2 - p_m^2)}\exp[i\int d^4x\frac{2}{\lambda}K^2]\frac{1}{\det[\frac{K}{V^2} + \frac{1}{2V}[2\frac{K_0}{V} - (m_0^2 - p_n^2)(\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2})]]^{1/2}}}{\int dK\exp[i\int d^4x\frac{2}{\lambda}K^2]\frac{1}{\det[\frac{K}{V^2} + \frac{1}{2V}[\frac{2K_0}{V} - (m_0^2 - p_n^2)(\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2})]]^{1/2}}} \tag{1.28}$$

Facem notatiile:

$$\frac{1}{\lambda}\frac{2}{V} = ba_0$$
$$m_0^2 - p_m^2 = c^2$$
$$\frac{2}{V} = a_0$$
$$\det[\frac{K}{V^2} + \frac{1}{2V}[\frac{2K_0}{V} - (m_0^2 - p_n^2)(\delta_{2n+1,2n+1} + \delta_{2n+2,2n+2})]] = \det[a_0K_0 + B] \tag{1.29}$$

Trebuie sa evaluam:

$$\frac{\int dK dK_0 \exp[2iba_0 K_0^2 + \int d^4x 2ibK^2] \frac{1}{(a_0K_0 - c^2)[\det[a_0K_0 + B]]^{1/2}}}{\int dK dK_0 \exp[ibK_0^2 + \int d^4x ibK^2)] \frac{1}{[\det[a_0K_0+B]]^{1/2}}} =$$

$$-\frac{1}{c^2}\frac{\int dK dK_0 \exp[2iba_0K_0^2 + \int d^4x 2ibK^2)][1 + \frac{a_0K_0}{c^2} + \frac{a_0^2K_0^2}{c^4} + ...]\frac{1}{[\det[a_0K_0+B]]^{1/2}}}{\int dK dK_0 \exp[ibK_0^2 + \int d^4x ibK^2]\frac{1}{[\det[a_0K_0+B]]^{1/2}}} \tag{1.30}$$

Pentru a determina raportul din Eq. (1.30) evaluam fiecare termen din dezvoltarea in serie din numarator:

$$\begin{aligned}
I_n = \int dK_0 dK \frac{(a_0K_0)^n}{c^{2n}} \exp[2iba_0K_0^2][\exp[\int d^4x 2ibK^2](\det[a_0K_0 + B])^{-1/2} = \\
\int dK_0 dK \frac{1}{a_0}\frac{d[\frac{(a_0K_0)^{n+1}}{c^{2n}(n+1)}]}{dK_0} \exp[2iba_0K_0^2][\exp[\int d^4x 2ibK^2](\det[aK_0 + B])^{-1/2} = \\
-\int dK_0 dK \frac{1}{a_0}\frac{(a_0K_0)^{n+1}}{c^{2n}(n+1)}(4iba_0K_0)\exp[2iba_0K_0^2]\exp[\int d^4x 2ibK^2](\det[aK_0 + B])^{-1/2} - \\
\int dK_0 dK \frac{(a_0K_0)^{n+1}}{a_0c^{2n}(n+1)}\exp[2iba_0K_0^2]\sum_k[\frac{-a_0}{a_0K_0 - c_k^2}][\exp[\int d^4x 2ibK^2](\det[a_0K_0 + B])^{-1/2} = \\
-\frac{4ibc^4}{a_0(n+1)}I_{n+2} - \int dK dK_0 \frac{(a_0K_0)^{n+1}}{c^{2n}(n+1)}\sum_k \frac{1}{c_k^2}[1 + \frac{a_0K_0}{c_k^2} + \frac{(a_0K_0)^2}{c_k^4} + ...] \times \\
\exp[2iba_0K_0^2]\exp[\int d^4x 2ibK^2](\det[a_0K_0 + B])^{-1/2}.
\end{aligned} \tag{1.31}$$

Din Eq. (1.88) se obtine urmatoarea formula de recurenta:

$$(n+1)I_n + I_{n+2}c^4[\frac{4ib}{a_0} + \sum_k \frac{1}{c_k^4}] + I_{n+1}c^2\sum_k \frac{1}{c_k^2} + ... + I_{n+r}c^{2r}\sum_k \frac{1}{c_k^{2r}} + ... = 0 \tag{1.32}$$

Se inmulteste intreaga Eq. (1.32) cu $\frac{1}{V}$, se face notatia $I_n c^{2n} = J_n$ si se obtine:

$$\frac{1}{V}(n+1)J_n + J_{n+1}\frac{1}{V}\sum_k \frac{1}{c_k^2} + J_{n+2}[2ib + \frac{1}{V}\sum_k \frac{1}{c_k^4}] + ... = 0 \tag{1.33}$$

In cele din urma se obtine din Eqs. (1.42) si (1.33) expresia pentru propagator:

$$\begin{aligned}
\text{Propagator} = -\frac{1}{c^2}\sum_n I_n/I_0 = \\
= -\frac{1}{c^2}\sum_n \frac{1}{c^{2n}} J_n/I_0.
\end{aligned} \tag{1.34}$$

Avem nevoie de urmatoarele notatii si rezultate utile:

$$\frac{1}{V}\sum_k \frac{1}{c_k^{2r}} = \frac{1}{V}\sum_k \frac{1}{(m_0^2 - p_k^2)^r} = (-1)^r \int d^4p \frac{1}{(p^2 - m_0^2)^r} = q_r, \tag{1.35}$$

si,

$$q_1 = i\frac{1}{16\pi^2}[\Lambda^2 - m_0^2 \ln[\frac{\Lambda^2}{m_0^2}]]$$
$$q_2 = i\frac{1}{16\pi^2}[-1 + \ln[\frac{\Lambda^2}{m_0^2}]]$$
$$q_{n,n>2} = i\frac{1}{16\pi^2}\frac{(m_0^2)^{2-n}}{(n-1)(n-2)}. \quad (1.36)$$

Expresia exacta pentru propagatorul unui camp scalar in teoria perturbatiilor este data de:

$$\frac{i}{p^2 - m^2 - M^2(p^2)} \quad (1.37)$$

unde m este masa fizica si $M^2(p^2)$ este "one particle irreducible self energy". In abordarea noastra propagatorul este:

$$\frac{i}{p^2 - m_0^2}\sum_n (-1)^n \frac{J_n}{I_0}\frac{1}{(p^2 - m_0^2)^n} \quad (1.38)$$

Dupa anumite calcule, simplificari se poate dovedi ca:

$$\frac{J_n}{I_0}|_{p^2=m^2} = [m_0^2 - m^2]^n. \quad (1.39)$$

Notam,

$$X = [m_0^2 - m^2], \quad (1.40)$$

si sumam in formula de recurenta din Eq. (1.33) toti termenii cu indicii $n+k$, $k \geq 3$ pentru $p^2 = m^2$.

$$\sum_{k\geq 3}\frac{J_{n+k}}{I_0}q_k = X^n \sum_k \frac{i}{16\pi^2}m_0^4(\frac{X}{m_0^2})^n \frac{1}{(n-1)(n-2)} =$$
$$\frac{i}{16\pi^2}X^{n+1}[X + (m_0^2 - X)\ln[\frac{m_0^2 - X}{m_0^2}]] \quad (1.41)$$

Formula de recurenta devine:

$$(n+1)a_0 X^n + q_1 X^{n+1} + (\frac{2i}{\lambda} + q_2)X^{n+2} + \frac{i}{16\pi^2}X^{n+1}[X + (m_0^2 - X)\ln[\frac{m_0^2 - X}{m_0^2}] = 0$$
$$(n+1)a_0\frac{1}{X} + q_1 + (\frac{2i}{\lambda} + q_2)X + \frac{i}{16\pi^2}[X + (m_0^2 - X)\ln[\frac{m_0^2 - X}{m_0^2}] = 0$$
$$q_1 + (\frac{2i}{\lambda} + q_2)X + \frac{i}{16\pi^2}[X + (m_0^2 - X)\ln[\frac{m_0^2 - X}{m_0^2}] = 0, \quad (1.42)$$

ceea ce conduce la:

$$q_1 + (m_0^2 - m^2)[\frac{2i}{\lambda} + q_2] + \frac{i}{16\pi^2}[(m_0^2 - m^2) + m^2 \ln[\frac{m^2}{m_0^2}]] = 0 \tag{1.43}$$

Eq. (1.43) determina masa fizica in functie de "bare mass" si de "cut-off scale". Pentru un "cut-off scale" mare se poate imparti Eq. (1.43) la q_1 si sa se retina numai primii doi termeni. Atunci,

$$m^2 \approx m_0^2 + \frac{q_1}{\frac{2i}{\lambda} + q_2} \approx m_0^2 + \frac{\Lambda^2 - m_0^2 \ln[\frac{\Lambda^2}{m_0^2}]}{1 + \frac{\lambda}{32\pi^2}[-1 + \ln[\frac{\Lambda^2}{m_0^2}]]}\frac{\lambda}{32\pi^2}. \tag{1.44}$$

E de mentionat ca acest rezultat conduce la acelasi coeficient de prim ordin la dimensiunea anomala a masei ca in procedurile de renormalizare standard. Urmatorii coeficienti sunt diferiti intrucat metoda utilizata constituie o procedura de renormalizare diferita.

C. Functia de partitite in teoriile de etalonare cu fermioni intr-un nou formalism functional

Cea mai simpla teorie de etalonare cu fermioni este QED cu o singura specie de fermioni, teorie care are Lagrangianul:

$$\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{1.45}$$

Scriem Eq. (1.45) in functie de modurile Fourier:

$$\int d^4x \mathcal{L}_{QED} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} A_\mu(k)[-k^2 g^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu]A_\nu(-k) + \\ + \int \frac{d^4k}{(2\pi)^4}[\Psi(k)(\gamma^\mu k_\mu - m)\Psi(-k)] - e\int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\bar{\Psi}(p)\gamma^\mu\Psi(-p+k)A_\nu(-k) \tag{1.46}$$

unde numaram peste modurile k positive sau negative.

Consideram expresia care contine campul de etalonare A_μ ca un termen quadratic plus un termen linear. Formam din acestia un patrat perfect si integram dupa modurile campului de etalonare A_μ:

$$W[0] = \int d\Psi d\bar{\Psi} dA_\mu \exp[i\int d^4x \mathcal{L}_{QED}] = \\ \text{const}\int \prod_i \prod_j d\Psi(p_i) d\bar{\Psi}(p_j)(\det[k^2 g^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu])^{-1/2} \times \\ \exp[i[\int \frac{d^4k}{(2\pi)^4}[\bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k)] - \\ \int e^2 \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}[\frac{1}{4}\bar{\Psi}(p)\gamma^\mu\Psi(p+k)D^{-1}_{\mu\nu}\bar{\Psi}(q)\gamma^\nu\Psi(q-k)]]. \tag{1.47}$$

Pentru a obtine aceasta expresie se face schimbarea de variabila $A_\mu(k) \rightarrow A_\mu(k) - \frac{\bar{\Psi}(p)\gamma^\nu\Psi(-p+k)}{2D^{\mu\nu}}$ si notatia :

$$D^{\mu\nu} = -k^2 g^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu, \tag{1.48}$$

unde ξ este parametrul de etalonare obisnuit. Vom integra in Eq. (1.47) prin introducerea unei noi variabile η_μ si a unei functii delta:

$$W[0] = \text{const} \int \prod_i \prod_j \prod_k d\bar{\Psi}(p_i) d\Psi(p_j) d\eta_\mu(p_k) (\det[k^2 g^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu])^{-1/2} \delta(\eta_\mu - \bar{\Psi}\gamma^\mu\Psi) \times$$
$$\exp[i[\int \frac{d^4k}{(2\pi)^4} \bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) - \int \frac{e^2}{4}\frac{d^4k}{(2\pi)^4} \eta^\mu D^{-1}_{\mu\nu}\eta^\nu]] \tag{1.49}$$

Mai departe exprimam functia delta in functie de reprezentarea ei exponentiala si obtinem:

$$W[0] = \text{const} \int \prod_i \prod_j \prod_k \prod_r d\bar{\Psi}(p_i) d\Psi(p_j) d\eta_\mu(p_k) dK_\mu(p_r) (\det[-i(k^2 g^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu)])^{-1/2} \times$$
$$\exp[i \int \frac{d^4k}{(2\pi)^4} K_\mu(p)(\eta_\mu(-p) - \bar{\Psi}(q)\gamma^\mu\Psi(-q-p))] \times$$
$$\exp[i[\int \frac{d^4k}{(2\pi)^4} \bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) - \int \frac{e^2}{4}\frac{d^4k}{(2\pi)^4} \eta^\mu(k) D^{-1}_{\mu\nu}\eta^\nu(-k)]]. \tag{1.50}$$

Integram peste variabila η_μ prin formarea unui patrat perfect in exponent si obtinem:

$$W[0] = \text{const} \int \prod_i \prod_j \prod_k d\bar{\Psi}(p_i) d\Psi(p_j) dK_\mu(p_k) (\det[-i(k^2 g^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu)])^{-1/2} \times$$
$$(\det[\frac{1}{e^2}D_{\mu\nu}])^{1/2} (\det[\frac{1}{e^2}D_{\mu\nu}])^{1/2} \exp[i \int \frac{d^4k}{(2\pi)^4}\frac{1}{2e^2} K_\mu(k) D^{\mu\nu}(k) K_\nu(-k)] \times$$
$$\exp[i \int \frac{d^4k}{(2\pi)^4} \bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) - i \int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4} K_\mu(k)\bar{\Psi}(p)\gamma^\mu\Psi(-k+p)] =$$
$$= \text{const} \int \prod_i dK_\mu(p_i) (\det[\frac{1}{2}(k^2 g^{\mu\nu} + (1 - \frac{1}{\xi})k^\mu k^\nu)])^{-1/2} (\det[\frac{2i}{e^2}D_{\mu\nu}])^{1/2} \times$$
$$\exp[i \int \frac{d^4k}{(2\pi)^4}\frac{1}{2e^2} K_\mu(k) D^{\mu\nu}(k) K_\nu(-k)] \times \det[(\gamma^\mu k_\mu - m)\delta_{m,n} - \gamma^\mu (K_\mu)_{-m-n=k}] \tag{1.51}$$

Este de mentionat ca rezultatul obtinut in Eq. (1.51) este exact acela pentru QED cu variabila A_μ inlocuita de noua variabila K_μ. Totusi procedura descrisa nu este redundanta intrucat prin pasii intermediari ne ajuta sa determinam functia beta a sarcinii electrice intr-o noua abordare.

Expresia pentru " two point function" este data de:

$$\int \prod_i \prod_j \prod_k dA_\mu(p_i) d\bar{\Psi}(p_j) d\Psi(p_k) A_\rho(p) A_\sigma(q) \exp[i \int d^4x \mathcal{L}_{QED}] \tag{1.52}$$

Facem din nou schimbarea de variabila $A_\nu(k) \to A_\nu(k) - \frac{e}{2}\Psi(p)\frac{\gamma^\nu}{D_{\mu\nu}}\Psi(p-k)$ si obtinem:

$$\begin{aligned}
I_{\rho\sigma} = & \int \prod_i \prod_j \prod_k dA_\mu(p_i) d\bar{\Psi}(p_j) d\Psi(p_k) [A_\rho(p) A_\sigma(q) + \frac{e^2}{4}\bar{\Psi}(r)\frac{\gamma^\mu}{D_{\mu\rho}(p)}\Psi(r+p)\bar{\Psi}(u)\frac{\gamma^\nu}{D_{\nu\sigma}(q)}\Psi(u+q)] \\
& \times \exp[i\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} A_\mu(k) D_{\mu\nu} A_\nu(-k) + \int \frac{d^4k}{(2\pi)^4}\bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) - \\
& \int e^2 \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}[\frac{1}{4}\bar{\Psi}(p)\gamma^\mu\Psi(p+k) D^{-1}_{\mu\nu}\bar{\Psi}(q)\gamma^\nu\Psi(q-k)]],
\end{aligned} \tag{1.53}$$

Intrucat integralele peste campurile de etalonare si " peste fermioni sunt separate putem scrie:

$$\begin{aligned}
I_{\rho\sigma} = & \int \prod_i dA_\mu(p_i)[A_\rho(p)A_\sigma(q)] \exp[i\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} A_\mu(k) D_{\mu\nu} A_\nu(-k)] \times \\
& \int \prod_j \prod_k d\bar{\Psi}(p_j) d\Psi(p_j) \exp[i \int \frac{d^4k}{(2\pi)^4}\bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) - \\
& \int e^2 \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}[\frac{1}{2}\bar{\Psi}(p)\gamma^\mu\Psi(p+k) D^{-1}_{\mu\nu}(k)\bar{\Psi}(q)\gamma^\nu\Psi(q-k)]] + \\
& \int \prod_i dA_\mu(p_i) \exp[i\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} A_\mu(k) D_{\mu\nu} A_\nu(-k)] \times \\
& \int \prod_j \prod_k d\bar{\Psi}(p_j) d\Psi(p_k)[\frac{e^2}{4}\bar{\Psi}(r)\frac{\gamma^\mu}{D_{\mu\rho}(p)}\Psi(r+p)\bar{\Psi}(u)\frac{\gamma^\nu}{D_{\nu\sigma}(q)}\Psi(u+q)] \times \\
& \exp[i \int \frac{d^4k}{(2\pi)^4}\bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) - \\
& \int e^2 \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}[\frac{1}{2}\bar{\Psi}(p)\gamma^\mu\Psi(p+k) D^{-1}_{\mu\nu}\bar{\Psi}(q)\gamma^\nu\Psi(q-k)]]
\end{aligned} \tag{1.54}$$

Primul termen in Eq. (1.54) poate fi separat ceea ce conduce la:

$$\begin{aligned}
& \int \prod_i dA_\mu(p_i)[A^\rho(k)A^\sigma(q)] \exp[i\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} A_\mu(k) D_{\mu\nu} A_\nu(-k)] = \\
& \delta(k-q)\frac{i\xi}{2}\frac{\delta^2}{\delta k_\rho \delta k_\sigma}(\det[i(-k^2 g^{\mu\nu} + (1-\frac{1}{\xi})k^\mu k^\nu])^{-1} = \\
& \frac{-i}{k^2}(g^{\rho\sigma} - \frac{k^\rho k^\sigma}{k^2})(\det[i(-k^2 g^{\mu\nu} + (1-\frac{1}{\xi})k^\mu k^\nu])^{-1},
\end{aligned} \tag{1.55}$$

ceea ce corespunde propagatorului campului de etalonare liber in etalonarea Landau.

Aici trebuie sa clarificam un punct. Operatorul quadratic kinetic care apare in Lagrangian este singular asa incat trebuie sa introducem parametrul de etalonare pentru consistenta. Vom lucra in etalonarea cu $\xi = 0$. Atunci,

$$i\xi \frac{\partial^2}{\partial k_\rho k_\sigma} i(-k^2 A_\mu(k)A^\mu(-k) + (1-\frac{1}{\xi})k^\mu k^\nu A_\mu(k)A_\nu(-k)) =$$
$$\xi[-2g^{\rho\sigma}A_\mu(k)A^\mu(-k) + 2(1-\frac{1}{\xi})A^\rho(k)A^\sigma(-k)] = A^\rho(k)A^\sigma(-k), \tag{1.56}$$

in limita $\xi = 0$.

Operatorul $D_{\mu\nu}$ satisface ecuatia:

$$[-k^2 g^{\mu\nu} + (1-\frac{1}{\xi})k^\mu k^\nu]\tilde{D}^{\nu\rho} = i\delta^\rho_\mu, \tag{1.57}$$

unde $\tilde{D}^{\nu\rho}$ este operatorul invers:

$$\tilde{D}^{\nu\rho} = \frac{-i}{k^2}(g^{\nu\rho} - (1-\xi)\frac{k^\nu k^\rho}{k^2}). \tag{1.58}$$

Se obtine in continuare:

$$\frac{\xi}{2}\frac{\delta^2}{\delta k^\rho \delta k^\sigma}\frac{1}{D_{\mu\nu}} = \frac{1}{D_{\mu\sigma}D_{\nu\rho}}. \tag{1.59}$$

de asemenea in etalonarea Landau. Aici sunt si alti termeni pe care ii ignoram pentru ca nu corespund diagramelor "one particle irreducible".

Mai departe calculam:

$$\int \prod_i \prod_j d\bar{\Psi}(p_i)d\Psi(p_j)[\frac{e^2}{4}\bar{\Psi}(r)\frac{\gamma^\mu}{D_{\mu\rho}(p)}\Psi(r+p)\bar{\Psi}(s)\frac{\gamma^\nu}{D_{\nu\sigma}(q)}\Psi(s+q)] \times$$
$$\exp[i\int \frac{d^4k}{(2\pi)^4}\bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) - \int e^2 \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}[\frac{1}{2}\bar{\Psi}(p)\gamma^\mu\Psi(p+k)D^{-1}_{\mu\nu}\bar{\Psi}(q)\gamma^\nu\Psi(q-k)]$$
$$= \frac{i\xi}{2}\frac{\partial^2}{\partial k^\rho k^\sigma}\int \prod_i \prod_j d\bar{\Psi}(p_i)d\Psi(p_j)\exp[i\int \frac{d^4k}{(2\pi)^4}\bar{\Psi}(k)(\gamma^\mu k_\mu - m)\Psi(-k) -$$
$$\int e^2 \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}[\frac{1}{2}\bar{\Psi}(p)\gamma^\mu\Psi(p+k)D^{-1}_{\mu\nu}\bar{\Psi}(q)\gamma^\nu\Psi(q-k)]], \quad (1$$

si observam ca trebuie sa consideram cantitatea:

$$\frac{i\xi}{2}\frac{\partial^2}{\partial k^\rho \partial k^\sigma}\mathrm{const}\int \prod_i dK_\mu(p_i)(\det[\frac{2i}{e^2}D_{\mu\nu}])^{1/2}(\det[iD_{\mu\nu}])^{-1/2} \times$$
$$\exp[i\int \frac{d^4k}{(2\pi)^4}\frac{1}{2e^2}K_\mu(k)D^{\mu\nu}K_\nu(-k)]\det[(\gamma^\mu p_\mu - m)\delta_{mn} - \gamma^\mu(K_\mu)_{-m-n=k}] = (\det[\frac{2}{e^2}])^{1/2} \times$$
$$\frac{i\xi}{2}\frac{\partial^2}{\partial k^\rho \partial k^\sigma}\exp[i\int \frac{d^4k}{(2\pi)^4}\frac{1}{2e^2}K_\mu(k)D^{\mu\nu}(k)K_\nu(-k)]\det[(\gamma^\mu p_\mu - m)\delta_{mn} - \gamma^\mu(K_\mu)_{-m-n=k}], \tag{1.61}$$

Apoi facem schimbarea de variabila $\frac{K_\mu}{e} \to K_\mu$ si obtinem:

$$I_{\rho\sigma} \approx \frac{i\xi}{2}\frac{\partial^2}{\partial k^\rho \partial k^\sigma}\int dK_\mu \exp[i\int \frac{d^4k}{(2\pi)^4}\frac{1}{2}K_\mu(k)D^{\mu\nu}K_\nu(-k)]\det[(\gamma^\mu p_\mu - m)\delta_{mn} - e\gamma^\mu (K_\mu)_{-m-n=k}] \tag{1.62}$$

Introducem operatorul,

$$O_{\alpha\mu} = \sqrt{\frac{i}{2k^2}}[k^2 g^{\mu\alpha} - (1-\frac{1}{\sqrt{\xi}})k^\mu k^\alpha] \tag{1.63}$$

care satisface relatia,

$$O_{\alpha\mu}O^\alpha_\nu = -\frac{i}{2}D_{\mu\nu}. \tag{1.64}$$

Facem din nou o schimbare de variabila:

$$K'_\mu = K_\alpha O^\alpha_\mu. \tag{1.65}$$

care conduce la:

$$I_{\rho\sigma} = \text{const}\frac{i\xi}{2}\frac{\partial^2}{\partial k^\rho \partial k^\sigma}\int \prod_i dK_\mu(p_i)(\det[i(k^2 g^{\mu\nu} - (1-\frac{1}{\xi})k^\mu k^\nu])^{-1}\exp[-\int \frac{d^4k}{(2\pi)^4}\frac{1}{2}K_\mu(k)K_\nu(-k)] \times$$
$$\det[(\gamma^\mu p_\mu - m)\delta_{mn} - \gamma^\mu e(K_\alpha)_{-m-n=k}(O^\alpha_\mu)^{-1}]. \tag{1.66}$$

Prima contributie vine de la:

$$\frac{i\xi}{2}\frac{\partial^2}{\partial k^\rho \partial k^\sigma}\det[k^2 g^{\mu\nu} - (1-\frac{1}{\xi})k^\mu k^\nu])^{-1} =$$
$$-i\frac{1}{k^2}(g^{\rho\sigma} - (1-\xi)\frac{k^\rho k^\sigma}{k^2})\det[i(k^2 g^{\nu\mu} - (1-\frac{1}{\xi})k^\mu k^\nu)])^{-1} \tag{1.67}$$

care este propgatorul campului liber. Pe langa aceasta urmatoarea contributie vine de la un termen de tipul:

$$\frac{\partial}{\partial k^\rho}(\det[k^2 g^{\mu\nu} - (1-\frac{1}{\xi})k^\mu k^\nu])^{-1}\frac{\partial}{\partial k^\sigma}\det[(\gamma^\mu p_\mu - m)\delta_{mn} - e\gamma^\mu (K_\alpha)_{-m-n=k}(O^\alpha_\mu)^{-1}], \tag{1.68}$$

care este zero in limita $\xi = 0$ dupa ce se face din nou schimbarea de variabila $K_\alpha \to -2io^\mu_\alpha K_\mu$.

Urmatoarea contributie este:

$$\frac{i\xi}{2}\frac{\partial^2}{\partial k^\rho \partial k^\sigma}\det[(\gamma^\mu p_\mu - m)\delta_{mn} - e\gamma^\mu (K_\alpha)_{-m-n=k}(O^\alpha_\mu)^{-1}] \tag{1.69}$$

In continuare pentru a determina functia beta avem nevoie de formula de diferentiere a unui determinant.

$$\frac{1}{2}\frac{\partial^2}{\partial k^\rho \partial k^\sigma}\det A = \frac{1}{2}\det A\text{Tr}[\frac{\partial A}{\partial k^\rho}A^{-1}]\text{Tr}[\frac{\partial A}{\partial k^\sigma}A^{-1}] +$$
$$\frac{1}{2}\det A\text{Tr}[\frac{\partial^2 A}{\partial k^\rho \partial k^\sigma}A^{-1}] - \frac{1}{2}\det A\text{Tr}[\frac{\partial A}{\partial k^\rho}A^{-1}\frac{\partial A}{\partial k^\sigma}A^{-1}]. \tag{1.70}$$

unde,

$$A=[(\gamma^{\mu}p_{\mu}-m)\delta_{mn}-e\gamma^{\mu}(K_{\alpha})_{-m-n=k}(O^{\alpha}_{\mu})^{-1}]. \tag{1.71}$$

Pentru a determina renormalizarea sarcinii data de $\frac{1}{1-\Pi(0)}$ si in consecinta functia beta trebuie sa calculam doi termeni:

$$\begin{aligned}&-i\xi\frac{1}{2}\det A\mathrm{Tr}[\frac{\partial A}{\partial k^{\rho}}A^{-1}\frac{\partial A}{\partial k^{\sigma}}A^{-1}]\\&i\xi\frac{1}{2}\det A\mathrm{Tr}[\frac{\partial A}{\partial k^{\rho}}A^{-1}]\mathrm{Tr}[\frac{\partial A}{\partial k^{\sigma}}A^{-1}],\end{aligned} \tag{1.72}$$

unde primul termen corespunde contributiei "one loop" si al doilea contributiei "two loops". Functia beta se opreste la "two loops".

Pentru a determina termeni de tipul $\frac{\delta A}{\delta k^{\rho}}$ mentionam ca prezenta factorului ξ in fata inseamna ca trebuie sa calculam numai contributiile proportionale cu $\frac{1}{\xi}$:

$$\frac{\delta A}{\delta k_{\rho}}=eK_{\alpha}\sqrt{\frac{2k^2}{i}}(1-\sqrt{\xi})\frac{1}{k^2}(g^{\mu\rho}\frac{k^{\alpha}}{k^2}+g^{\alpha\rho}\frac{k^{\mu}}{k^2}) \tag{1.73}$$

Apoi facem un nou schimb de variabile: $K^{\alpha}=K'_{\nu}\frac{k^2g^{\nu\alpha}-(1-\frac{1}{\sqrt{\xi}})k^{\nu}k^{\alpha}}{\sqrt{k^2}}$ si determinam:

$$eK_{\alpha}\sqrt{\frac{2k^2}{i}}(1-\sqrt{\xi})\frac{1}{k^2}(g^{\mu\rho}\frac{k^{\alpha}}{k^2}+g^{\alpha\rho}\frac{k^{\mu}}{k^2})\to e\sqrt{\frac{2}{i}}\frac{1}{k^2}\frac{1}{\sqrt{\xi}}K'_{\nu}k^{\nu}g^{\mu\rho} \tag{1.74}$$

unde consideram doar termenul proportional cu $\frac{1}{\sqrt{\xi}}$.

Prima contributie in Eq. (1.76) este obtinuta astfel,

$$\begin{aligned}&-i\xi\frac{1}{2}\det A\mathrm{Tr}[\frac{\partial A}{\partial k^{\rho}}A^{-1}\frac{\partial A}{\partial k^{\sigma}}A^{-1}]=\\&-\frac{i}{2(k^2)^2}\frac{2}{i}\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-m^2)^2}\mathrm{Tr}e^2[k^{\alpha}K_{\alpha}k^{\beta}K_{\beta}\gamma^{\rho}(\gamma^{\tau}p_{\tau}+m)\gamma^{\sigma}(\gamma^{\eta}p_{\eta}+m)]=\\&=-ig^{\rho\sigma}\frac{1}{k^2}\frac{2}{64\pi^2}[-\Lambda^2+4m^2\ln[\frac{\Lambda^2}{m^2}]]K^2_{\mu}e^2.\end{aligned} \tag{1.75}$$

Rezultatul trebuie impartit la functia de partitie ceea ce conduce la:

$$-e^2\frac{2}{64\pi^2}[\Lambda^2-4m^2\ln[\frac{\Lambda^2}{m^2}]]\frac{\int dK_{\mu}K^2_{\mu}}{\int dK_{\mu}}=-e^2\frac{1}{16\pi^2}\frac{1}{3}[1-4\frac{m^2}{\Lambda^2}\ln[\frac{\Lambda^2}{m^2}]], \tag{1.76}$$

unde integrala peste variabila K este realizata in coordonate sferice in spatiul euclideean cu un "cut-off" Λ.

Termenul "two loop" este calculat astfel:

$$i\xi\frac{1}{2}\det A\mathrm{Tr}[\frac{\partial A}{\partial k^{\rho}}A^{-1}]\mathrm{Tr}[\frac{\partial A}{\partial k^{\sigma}}A^{-1}]=ig^{\rho\sigma}\frac{1}{k^2}\frac{1}{3}\frac{1}{64\pi^4}(-\Lambda^2+3m^2\ln[\frac{\Lambda^2}{m^2}])^2(K^2_{\mu})^2 \tag{1.77}$$

Ecuatia de mai sus se imparte la functia de partitie si se obtine:

$$-e^4\frac{1}{64\pi^4}(-\Lambda^2+3m^2\ln[\frac{\Lambda^2}{m^2}])^2\frac{\int dK_\mu (K_\mu^2)^2}{\int dK_\mu} = -e^4\frac{1}{64\pi^4}(-1+3\frac{m^2}{\Lambda^2}\ln[\frac{\Lambda^2}{m^2}])^2\frac{1}{2}. \quad (1.78)$$

In final rezulta functia beta pentru sarcina electrica:

$$\beta(\alpha) = \frac{\partial(\frac{\alpha}{\pi})}{\partial\ln[M^2]} = \frac{1}{3}(\frac{\alpha}{\pi})^2 + \frac{1}{4}(\frac{\alpha}{\pi})^4, \quad (1.79)$$

si are numai primii doi coeficienti diferiti de zero ceea ce corespunde la renormalizarea 't Hooft intr-o noua abordare functionala.

D. Functia de partitite in teoriile de etalonare cu interactie tare intr-un nou formalism functional

Urmatorul Lagrangian pe care il vom studia este Lagrangianul Yang Mills:

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2, \quad (1.80)$$

unde,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu. \quad (1.81)$$

Lagrangianul din Eq. (1.80) este fixat prin introducerea Lagrangianului "ghost":

$$\mathcal{L}_g = \bar{c}^a(-\partial^\mu\partial_\mu - g f^{abc}\partial^\mu A^b_\mu)c^c. \quad (1.82)$$

Vom lucra in etalonarea Feynman ($\xi = 1$) si in spatiul Fourier. Utilizand,

$$A^a_\mu(x) = \frac{1}{V}\sum_n \exp[-ik_n x]A^a_\mu(k_n), \quad (1.83)$$

Lagrangianul poate fi rescris ca:

$$\begin{aligned}
\int d^4x\mathcal{L} = &-\frac{1}{2}\frac{1}{V}\sum_n k_n^2 A^{a\nu}(k_n)A^a_\nu(-k_n) + \frac{1}{V}\sum_n k_n^2\bar{c}^a(k_n)c^a(-k_n) + \\
&+\frac{i}{V^2}\sum_{n,m} k_n^\mu A^a_\nu(k_n) f^{abc} A^b_\mu(k_m) A^{c\nu}(-k_n-k_m) - \\
&-\frac{1}{V^3} f^{abc} f^{ade}\sum_{n,m,p} A^{b\mu}(k_n)A^{c\nu}(k_m)A^d_\mu(k_p)A^e_\nu(-k_n-k_m-k_p) - \\
&-\frac{i}{V^2}\sum_{n,m} k_n^\mu \bar{c}^a(k_n) g f^{abc} A^b_\mu(k_m) c^c(-k_n-k_m). \quad (1.84)
\end{aligned}$$

Functia de partitie este definita de:

$$Z_0 = \int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) \exp[i \int d^4x \mathcal{L}], \tag{1.85}$$

unde in exponent trebuie utilizata Eq. (1.84).

Aceasta poate fi rescrisa ca:

$$Z_0 = \text{factor} \times \exp[\sum_i V_i] \tag{1.86}$$

unde V_i este o diagrama tipica "disconnected". Toate diagramele V_i diagrams sunt inchise si nu depind de impulsuri. Factorul din fata este un produs obtinut prin inetegrarea integralelor de tip "gaussian" care corespund termenilor kinetici. Asadar expresia completa pentru functia de partitie este:

$$Z_0 = \text{const} \prod_i (k_i^2)^{N^2-1} \prod_j (k_j^2)^{-d/2(N^2-1)} \exp[\sum_i V_i] \tag{1.87}$$

unde N vine de la grupul Yang Mills $SU(N)$.

Mai departe calculam:

$$\begin{aligned}
Z_0 &= \int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) \exp[i \int d^4x \mathcal{L}] = \\
&= \int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) \frac{dA_\nu^a(k)}{dA_\nu^a(k)} \exp[i \int d^4x \mathcal{L}] = \\
&= \int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) \frac{d}{dA_\nu^a(k)} [A_\nu^a(k) \exp[i \int d^4x \mathcal{L}]] - \\
&- \int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) A_\nu^a(k) \frac{d}{dA_\nu^a(k)} \exp[i \int d^4x \mathcal{L}].
\end{aligned} \tag{1.88}$$

Mai intai analizam primul termen dupa semnul egal din Eq. (1.88). Obtinem:

$$\begin{aligned}
&\int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) A_\nu^a(k) \exp[i \int d^4x \mathcal{L}]_{A_\nu^a(k)=+\infty} - \\
&\int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) A_\nu^a(k) \exp[i \int d^4x \mathcal{L}]_{A_\nu^a(k)=-\infty}.
\end{aligned} \tag{1.89}$$

Factorul exponential in Eq. (1.89) va contine:

$$\exp[i \int d^4x \mathcal{L}] \sim \text{other factors} \times \exp[-\frac{i}{2} k^2 A^{a\nu}(k) A_\nu^a(k)] \tag{1.90}$$

Cantitatea k^2 trebuie de fapt sa fie scrisa ca $k^2 + i\epsilon$ unde ϵ asigura convergenta integralei gaussiene. Atunci se obtine:

$$\lim_{A^a_\nu \to \pm\infty} A^a_\nu(k) \exp[-\frac{i}{2}k^2 A^{a\nu}(k)A^a_\nu(k) - \frac{\epsilon}{2}A_{a\nu}(k)A^a_\nu(k)] = 0 \tag{1.91}$$

Asadar prima contributie dupa semnul egal in Eq. (1.88) este zero. Urmatoarea contributie este:

$$Z_0 = \int \prod_i \prod_j \prod_m dA^a_\mu(k_i)d\bar{c}^b(k_j)dc^d(k_m)(-i)[-\frac{k^2}{V}A^{a\nu}(k)A^a_\nu(-k) +$$
$$\frac{3i}{V^2}gk^\mu \sum_p f^{abc}A^a_\nu(k)A^b_\mu(p)A^{c\nu}(-k-p) - \frac{i}{V^2}g\sum_p p^\nu \bar{c}^b(p)f^{bac}A^a_\nu(k)c^c(-p-k) -$$
$$-\frac{1}{V^3}g^2 f^{bac}f^{bde}\sum_{p,q} A^a_\nu(k)A^c_\mu(p)A^{d\nu}(q)A^{e\mu}(-p-k-q)] \times \exp[i\int d^4x\mathcal{L}]. \tag{1.92}$$

In conformitate cu Eq. (1.87) se poate scrie:

$$k^\mu \frac{dZ_0}{\partial k^\mu} = -2(N^2-1)[\frac{d}{2}-1]Z_0 \tag{1.93}$$

Aplicam operatorul $k^\mu \frac{d}{dk^\mu}$ ecuatiei Eq. (1.85) si obtinem:

$$k^\mu \frac{dZ_0}{dk^\mu} = \int \prod_i \prod_j \prod_m dA^a_\mu(k_i)d\bar{c}^b(k_j)dc^d(k_m) \times$$
$$i[-\frac{1}{V}k^2 A^{a\nu}(k)A^a_\nu(-k) + \frac{2}{V}k^2\bar{c}^a(k)c^a(-k) + \frac{i}{V^2}k^\mu \sum_p A^a_\nu(k)f^{abc}gA^b_\mu(p)A^{c\nu}(-p-k) -$$
$$-\frac{i}{V^2}\sum_p k^\mu \bar{c}^a(k)gf^{abc}A^b_\mu(p)c^c(-p-k)] \times \exp[i\int d^4x\mathcal{L}. \tag{1.94}$$

Urmatorul pas este sa consideram Lagrangianul Yang Mills din perspectiva renormalizarii ceea ce conduce la:

$$\mathcal{L}_r = -\frac{1}{2}\frac{1}{V}Z_3\sum_n k_n^2 A^{a\nu}(k_n)A^a_\nu(-k_n) + \frac{1}{V}Z_2\sum_n k_n^2\bar{c}^a(k_n)c^a(-k_n) +$$
$$+\frac{i}{V^2}Z_{3g}\sum_{n,m} k_n^\mu A^a_\nu(k_n)f^{abc}A^b_\mu(k_m)A^{c\nu}(-k_n-k_m) -$$
$$-\frac{1}{V^3}Z_{4g}g^2 f^{abc}f^{ade}\sum_{n,m,p} A^{b\mu}(k_n)A^{c\nu}(k_m)A^d_\mu(k_p)A^e_\nu(-k_n-k_m-k_p) -$$
$$-\frac{i}{V^2}Z'_1\sum_{n,m} k_n^\mu \bar{c}^a(k_n)gf^{abc}A^b_\mu(k_m)c^c(-k_n-k_m). \tag{1.95}$$

Aici campurile de etalonare si cuplajele trebuie considerate renormalizate si constantele de renormalizare satisfac identitatile Slanov-Taylor:

$$g_0^2 = \frac{Z_{3g}^2}{Z_3^3} g^2 \mu^\epsilon = \frac{Z_{4g}}{Z_3^2} g^2 \mu^\epsilon = \frac{Z_1'^2}{Z_2'^2 Z_3} g^2 \mu^\epsilon, \tag{1.96}$$

unde $d = 4 - \epsilon$ si μ este un parametru cu dimensiunea de masa.

In metoda "background gauge field" care consta in separarea campului de etalonare A_μ^a intr-un "background" camp de etalonare B_μ^a si o fluctuatie cuantica $\tilde{A}_\mu^a$ se obtin relatii mai simple intre constantele de renormalizare:

$$\begin{aligned} Z_{4g} &= Z_{3g} = Z_3 \\ Z_2 &= Z_1' \\ Z_g &= Z_3^{-1/2}. \end{aligned} \tag{1.97}$$

Aplicate la Lagrangianul renormalizat Eqs.(1.92) si (1.94) vor deveni:

$$\begin{aligned} Z_0 = &\int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) (-i)[-\frac{k^2}{V} Z_3 A^{a\nu}(k) A_\nu^a(-k) + \\ &\frac{3i}{V^2} Z_{3g} g k^\mu \sum_p f^{abc} A_\nu^a(k) A_\mu^b(p) A^{c\nu}(-k-p) - \frac{i}{V^2} g Z_1' \sum_p p^\nu \bar{c}^b(p) f^{bac} A_\nu^a(k) c^c(-p-k) - \\ &-\frac{1}{V^3} g^2 Z_{4g} f^{bac} f^{bde} \sum_{p,q} A_\nu^a(k) A_\mu^c(p) A^{d\nu}(q) A^{e\mu}(-p-k-q)] \times \exp[i \int d^4x \mathcal{L}], \end{aligned} \tag{1.98}$$

si

$$\begin{aligned} &-2(N^2-1)[\frac{d}{2} - 1] Z_0 = \int \prod_i \prod_j \prod_m dA_\mu^a(k_i) d\bar{c}^b(k_j) dc^d(k_m) \times \\ &i[-\frac{1}{V} Z_3 k^2 A^{a\nu}(k) A_\nu^a(-k) + \frac{2}{V} Z_2 k^2 \bar{c}^a(k) c^a(-k) + \frac{i}{V^2} k^\mu Z_{3g} \sum_p A_\nu^a(k) f^{abc} g A_\mu^b(p) A^{c\nu}(-p-k) - \\ &-\frac{i}{V^2} Z_1' \sum_p k^\mu \bar{c}^a(k) g f^{abc} A_\mu^b(p) c^c(-p-k)] \times \exp[i \int d^4x \mathcal{L}]. \end{aligned} \tag{1.99}$$

Aplicam formula de reductie LSZ ecuatiilor (1.98) si (1.99) si obtinem:

$$\begin{aligned} 1 = &a_1 Z_3 + a_2 g Z_{3g} \sum_p k^\mu \frac{1}{k^2 p^2 (p+k)^2} f^{abc} \langle \vec{k}, \epsilon_{a,\nu}; \vec{p}, \epsilon_{b,\mu} | S | \overrightarrow{p+k}, \epsilon_{c,\nu} \rangle + \\ &+ a_2 g Z_1' \sum p^\mu \frac{1}{p^2 k^2 (p+k)^2} f^{abc} \langle \vec{p}, a; \vec{k}, \epsilon_{b,\mu} | S | \overrightarrow{p+k}, c \rangle + \\ &+ a_3 g^2 Z_{4g} f^{bac} f^{bde} \langle \vec{k}, \epsilon_{a,\nu}; \vec{p}, \epsilon_{c,\mu} | S | -\vec{q}, \epsilon_{d,\nu}; \overrightarrow{k+p+q}, \epsilon_{e,\mu} \rangle, \end{aligned} \tag{1.100}$$

si,

$$-2(N^2-1)(\frac{d}{2}-1) = b_1 Z_3 + b_2 Z_2' + b_3 Z_{3g} \sum_p k^\mu \frac{1}{k^2 p^2 (p+k)^2} f^{abc} \langle \vec{k}, \epsilon_{a,\nu}; \vec{p}, \epsilon_{b,\mu} | S | \overrightarrow{p+k}, \epsilon_{c,\nu} \rangle +$$

$$b_4 g Z_1' \sum k^\mu \frac{1}{p^2 k^2 (p+k)^2} f^{abc} \langle \vec{k}, a; \vec{p}, \epsilon_{b,\mu} | S | \overrightarrow{p+k}, c \rangle. \tag{1.101}$$

Eqs. (1.100) and (1.101) contin relatii intre constantele de renormalizare si functiile "two", "three" si "four point". Utilizand,

$$a_2 g Z_{3g} g \sum_p k^\mu \frac{1}{k^2 p^2 (p+k)^2} f^{abc} \langle \vec{k}, \epsilon_{a,\nu}; \vec{p}, \epsilon_{b,\mu} | S | \overrightarrow{p+k}, \epsilon_{c,\nu} \rangle =$$

$$a_2 Z_{3g} g \sum_p k^2 \frac{1}{k^2 p^2 (p+k)^2} f^{abc} f^{abc} \Gamma(p, k, -(p+k)) + ... \tag{1.102}$$

si faptul ca p^2, k^2, pk sunt "on-shell" se obtine:

$$a_2 Z_{3g} \sum_p k^2 \frac{1}{k^2 p^2 (p+k)^2} f^{abc} f^{abc} \Gamma(p, k, -(p+k)) = a_2 Z_{3g} \sum_p \frac{1}{p^2 (p+k)^2} \mathrm{const} g^2 =$$

$$a_2 Z_{3g} g^2 \mathrm{const} \frac{(p^2)^2}{p^(k+p)^2} = b Z_{3g} g^2 \tag{1.103}$$

Aplicand aceeasi procedura ecuatiilor (1.100) si (1.101) se obtine:

$$1 = a Z_2 + b Z_{3g} g^2 + c Z_1'^2 g^2 + d Z_{4g} g^4, \tag{1.104}$$

si

$$x = y Z_3 + z Z_2 + u Z_{3g} g^2 + w Z_1' g^2. \tag{1.105}$$

Aici a, b, c, d, x, y, z, u, w sunt constante independende de constanta de cuplaj dar care raman nedeterminate.

Prin acelasi procedeu aplicat campurilor "ghost" se obtine usor o noua relatie:

$$r_1 = r_2 Z_2 + r_3 Z_1' g^2, \tag{1.106}$$

unde r_1, r_2 si r_3 sunt constante independende de constanta de cuplaj dar care raman nedeterminate.

In abordarea standard nu se pot determina constantele de renormalizare din ecuatiile (1.104), (1.105) si (1.106) intrucat avem trei ecuatii si cinci constante de renormalizare. In metoda "background gauge field" lucrurile se simplifica si ecuatiile (1.104), (1.105) si (1.106) conduc la:

$$
\begin{aligned}
1 &= (f_1 + f_2 g^2 + f_3 g^4) Z_3 + f_4 Z_2 g^2 \\
1 &= (h_1 + h_2 g^2) Z_3 + h_4 Z_2 g^2 \\
1 &= (c_1 + c_2 g^2) Z_2
\end{aligned}
\tag{1.107}
$$

unde f_i, h_i si c_i sunt constante, unele din ele divergente. De aici se determina o formula pentru Z_3 si o conditie de consistenta:

$$
\begin{aligned}
Z_3 &= \frac{c_1 + (c_2 - h_4) g^2}{(c_1 + c_2 g^2)(h_1 + h_2 g^2)} \\
\frac{c_1 + (c_2 - h_4) g^2}{h_1 + h_2 g^2} &= \frac{c_1 + (c_2 - f_4) g^2}{f_1 + f_2 g^2 + f_3 g^4}
\end{aligned}
\tag{1.108}
$$

Din a doua ecuatie se determina printre altele $c_2 = h_4$ si din prima Z_3:

$$
Z_3 = \frac{1}{1 + d_1 g^2 + d_2 g^4},
\tag{1.109}
$$

unde $d_1 = h_2 + h_1 c_2 / c_1$, $d_2 = c_2 / c_1 h_2$ si $h_1 = 1$.

Vom utiliza dezvoltarea in serie a constantei de renormalizare Z_3 in functie de parametrul de regularizare dimensionala $\frac{1}{\epsilon}$ desi procedura noastra de renormalizare nu este identica cu renormalizarea dimensionala:

$$
Z_1 = 1 + \frac{Z_3^{(1)}}{\epsilon} + \frac{Z_3^{(2)}}{\epsilon^2} + \ldots
\tag{1.110}
$$

Coeficientii d_1 si d_2 sunt divergenti si pot fi dezvoltati in serie in functie de $\frac{1}{\epsilon}$:

$$
\begin{aligned}
d_1 &= \frac{d_{(1)}}{\epsilon} + \frac{d_1^{(2)}}{\epsilon^2} + \ldots \\
d_2 &= \frac{d_2^{(2)}}{\epsilon} + \frac{d_2^{(2)}}{\epsilon^2} + \ldots
\end{aligned}
\tag{1.111}
$$

Aplicand Eqs. (1.110) si (1.111) ecuatiei (1.109) se obtine:

$$
Z_1^{(1)} = -d_1^{(1)} g^2 - d_2^{(1)} g^4
\tag{1.112}
$$

Functia beta este definita ca:

$$
\beta = \mu^2 \frac{dg^2}{d\mu^2} = -g^4 \frac{\partial Z_3^{(1)}}{\partial g^2} = g^4 (d_1^{(1)} + g^2 d_2^{(2)})
\tag{1.113}
$$

Intrucat primii doi coeficienti sunt universali functia beta este determinata complet si corespunde schemei't Hooft. Coeficientii $d_1^{(1)}$ si $d_2^{(1)}$ sunt identificati cu:

$$
\begin{aligned}
d_1^{(1)} &= -\frac{11}{3}N\frac{1}{(4\pi)^2} \\
d_2^{(1)} &= -\frac{34}{3}N^2\frac{1}{(4\pi)^4}.
\end{aligned}
\tag{1.114}
$$

II. LUCRARI RAPORTATE IDEI PENTRU PERIOADA SEPTEMBRIE 2013-DECEMBRIE 2014

1. "About the role of scalars in a gauge theory", Renata Jora, Salah Nasri, arXiv: 1310.6122, in evaluare.

2. "A hierarchy of the quark masses in a top condensate model with multiple Higgses", Amir H. Fariborz, Renata Jora, Salah Nasri, Joseph Schechter, arXiv:1310.1721, Modern Physics Letters A 29, 6, 1450030 (2014).

3. "A nonperturbative method for the scalar field theory", R. Jora, arXiv:1403.2227 (2014), in evaluare.

4. "A hint of a strong supersymmetric standard model", R. Jora and J. Schechter, arXiv:1403.3778 (2014), acceptata de Int. J. Mod. Phys. A.

5. "A semi perturbative method for QED", R. Jora and J. Schechter, arXiv:1407.8172 (2014), in evaluare.

6. "Tree level relations in a composite electroweak theory", A. H. Fariborz, R. Jora and J. Schechter, arXiv:1407.6546 (2014), Mod. Phys. Lett. A 29, 31, 1450166.

7. "Probing scalar mesons in semi-leptonic decays of D_s^+, D^+ and D^0", A. H. Fariborz, R. Jora, J. Schechter, M.N. Shahid, arXiv:1407.7176 (2014), acceptata de Int. J. Mod. Phys. A.

8. "A semiclassical approach for the Higgs boson", A.H. Fariborz, R. Jora, J. Schechter, arXiv:1409.7886 (2014), acceptata de Int. J. Mod. Phys. A.

9. "A nonperturbative method for the Yang Mills Lagrangian", R. Jora, arXiv:1411.0211 (2014), in evaluare.

In consecinta am realizat toate activitatile pe anii 2013, 2014.

Consider ca activitatile preconizate in cadrul propunerii de proiect au fost implementate intr-o mare masura si ca realizarea acestora va fi finalizata cu succes pana la sfarsitul derularii proiectului.

Director de Proiect

Catalina Renata Jora